Tidal forces in the Solar System

Introduction

As anywhere else in the Universe, gravity is the basic and fundamental principle that rules the shape and permanent motion of all the celestial bodies inside the Solar System. No matter the type of planet, moon, asteroid or comet, gravity accounts for the main parameters of each orbit, which can be calculated by considering each body as a point mass.

But there are other gravitational effects, those coming from the fact that actual bodies do have internal dimensions, which become the major cause for specific and unique characteristics of a lot of members of the Solar System. Such effects are derived from differential gravitational forces or "tidal forces", which not only have dramatically affected the development of the present state of the Solar System, but will endlessly continue reshaping it as well.

Tidal forces fundamentals

According to Newton's law of universal gravitation, the attractive force (F) reciprocally exerted between two masses (m, M) being separated at a distance (d) is:

$$F = \frac{GMm}{d^2}$$

where G is the universal constant of gravitation.

Inside a celestial body, the gravitational force exerted by the distant mass (M) on any small element of mass (m) varies as its relative position changes, having an overall distribution as illustrated in Figure 1.

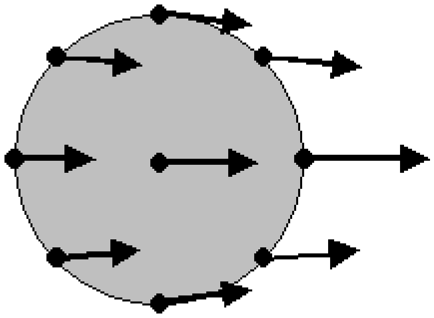


Figure 1
Forces relative to the distant mass

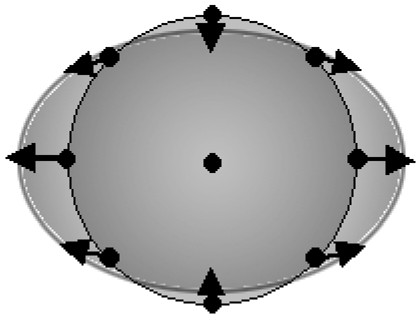


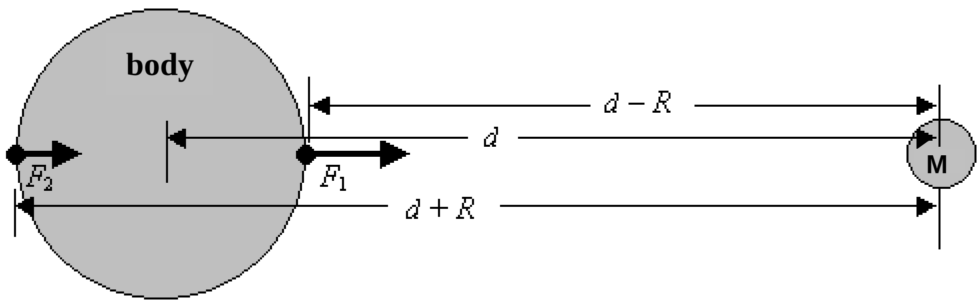
Figure 2
Forces relative to the center

Subtracting everywhere the central force vector from each local force vector, it's obtained the distribution of forces relative to center of the body, as Figure 2 shows.

Those "differential" forces stretch the body laterally and compress it in the perpendicular direction, distorting it from the spherical shape into a prolate spheroid (rugby ball shape) as two "tidal bulges" at each side from the distant mass (M) have appeared. [1]

Technically named "differential gravitational forces", they are normally called "tidal forces" because of their well-known effect producing diurnal ocean tides every 12 hours in the spinning Earth due to the Moon.

The greater difference "tidal" force (ΔF) value comes from two elements of mass (m) at each side of the body, diametrically separated over the same line to the gravitational mass (M), as seen in the following diagram:



$$F_2 = \frac{GMm}{(d+R)^2} \qquad F_1 = \frac{GMm}{(d-R)^2} \qquad \Delta F = F_2 - F_1 = \frac{GMm}{d^2}\left[\left(1+\frac{R}{d}\right)^{-2} - \left(1-\frac{R}{d}\right)^{-2}\right]$$

For the usual case of $d \rangle\rangle R$, the greater tidal force value results:

$$\Delta F \approx \frac{4GMmR}{d^3}$$

where R is the radius of the body
d is the distance from the center of the body to the gravitational mass M

As the distance to the gravitational mass increases, the value of the tidal force rapidly decays due to result proportional to $1/d^3$. Conversely, if the distance to the gravitational mass becomes near enough, it could be that the disruptive tidal forces even overcome the proper cohesive gravitational forces that hold the body together, making it break up into fragments.

Therefore, for each planet there is a minimum safe distance from its center in order that any natural satellite could revolve it without risk of being disintegrated. That minimum distance is the "Roche limit".

Consequences of tidal forces in the Solar System

a) Earth-Moon system

Tidal bulges on Earth are basically due to the Sun and the Moon. Because of its nearness, tidal forces from the Moon are about twice as great as those from the Sun, which accounts for the noticeable direct effect of the Moon on Earth's ocean tides. As the body of the Earth is largely rigid, it cannot deform very much in response to tidal forces, as water in the oceans can actually do.

The highest -"spring"- tides are obtained when tidal bulges from the Sun and the Moon overlap (as in full or new Moon case), and the smallest -"neap"- tides occur when they are at right angle (first or last quarter case).

Because the Earth rotates on its axis more rapidly than the Moon revolves around it, the tidal bulges are not exactly aligned in the Earth-Moon direction, becoming dragged ahead as seen in Figure 3. That makes that the high tide appears delayed everywhere with respect to the time of local highest moon.

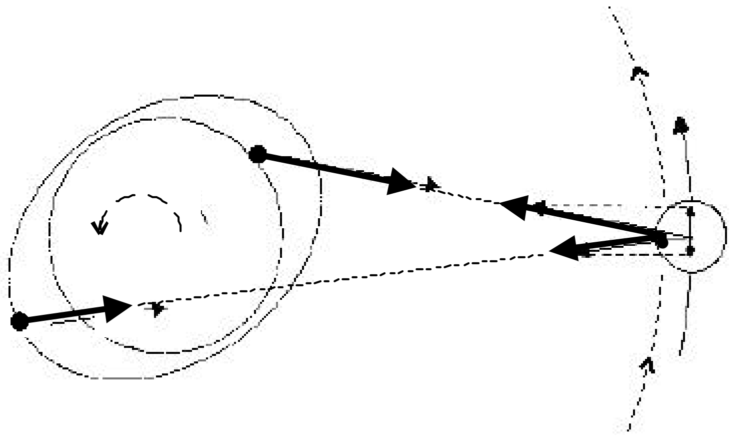


Figure 3
Misalignment of Earth's tidal bulges and net force exerted on the Moon's orbital direction

That misalignment also implies that the Moon becomes affected by Earth's tidal forces, as it appears a non-zero net force in the same direction of the Moon's orbital motion which tends to move it up and away from the Earth, as Figure 3 also shows.

The Earth's daily tides (mostly oceanic but also on solid surfaces) cause loss of energy of its rotation due to friction, meaning that the Earth is slowing down its rotation rate (currently about 0.0016 seconds per century). At the same time the Moon is moving away (about 3.8 meters per century), so its orbital period increases. This process will go on until a stable situation could be reached when the rotation period of the Earth equals the revolving period of the Moon. In such scenario, tidal forces by the Moon on the Earth will remain steady because the Earth will be "tidally locked" to the Moon, always keeping the same face towards it.

The Moon was tidally locked to the Earth a long time ago because tidal forces of Earth on the Moon are much stronger.

Once the Earth-Moon becomes tidally locked, only other external tidal forces will affect its configuration. As solar tides will tend to slow the Earth's rotation even more, the tidal bulges due to the Moon will begin to get behind (in the reverse inclination as seen in Fig. 3). Therefore there would appear a net force opposite to the Moon's orbital motion, forcing it to spiral inward until it's disintegrated on reaching the Earth's Roche limit (18,500 km) [2].

b) Precession, nutation and climate changes

The Earth is slightly oblate because of its rotation. As the Earth presents a tilted "equatorial bulge" to both the Sun and the Moon, differential forces appear trying to set the Earth's axis of rotation upright. Because the Earth is spinning, those tidal forces actually change the angular momentum perpendicular to the rotation, causing the Earth to precess, that is, its axis of rotation slowly traces out a circle in some 26,000 years, while keeping about the same inclination to the ecliptic ("obliquity").

Due to the Moon's orbit being slightly tilted from the ecliptic, the Earth's obliquity also gradually changes as the rotational axis wobbles while precessing. This slight nodding of the axis is called "nutation", forcing the precession motion to actually become a wiggly circle in the sky [3].

Tidal forces from the Sun and the Earth affect the orbit of the Moon, causing similar effects as precession and nutation on its orbit.

Cyclical changes in the Earth's obliquity and eccentricity, ultimately due to the tidal forces of Jupiter and the other planets, account for some climate changes that have happened, such as the advances and retreats of continental glaciers. In the same way, others planets tidally affected like Mars *"may be expected to have similar climate fluctuations"* [4].

c) Synchronous rotation

Many objects in the Solar System are in synchronous rotation, so that their rotational period has an exact numerical relation with their period of revolution (i.e. the Moon, having both the same). This is a direct consequence of tidal forces generated by a central object inside the revolving rotational body, having acted over a sufficiently long period of time.

Having reached the synchronous rotation condition, most of the time the body has equal rotation and revolution periods (so called “1-to-1 spin-orbit coupling”) meaning that the body always shows the same face to the central object, revolving in a nearly circular orbit. That is the current situation for 23 satellites in the Solar System (the Earth's Moon, Mars's Phobos and Deimos, Jupiter's Amalthea, Io, Europa, Ganymede and Callisto, Saturn's, Janus, Epimetheus, Mimas, Enceladus, Tethys, Dione, Rhea and Iapetus, Uranus' Miranda, Ariel, Umbriel, Titania and Oberon, Neptune's Triton, and Pluto's Charon). Another 7 satellites are also suspected of being in synchronous rotation: Jupiter's Thebe, Saturn's Prometheus, Pandora, Calypso and Titan, Neptune's Larissa and Proteus [5].

One special case of synchronous rotation is Mercury, which is locked into a 3-to-2 spin-orbit coupling as a direct consequence of its rather eccentric orbit (e = 0.206). Instead of always keeping the same face towards the Sun, as it would be in the case of a nearly circular orbit, Mercury makes three complete rotations for every two complete orbits.

Pluto has already been synchronized by its previous tidally locked moon Charon, therefore resulting in the first tidally locked system in the whole Solar System.

Synchronous rotation doesn't mean that the orbital parameters are stable. Tidal forces induced by satellites on theirs parent planets will continually reshape each moon's orbit until reciprocal -tidally locked- stable conditions can be achieved.

Both Mars's Phobos and Deimos are in synchronous rotation. But while Deimos is slowly moving away from Mars, Phobos is slowly spiraling towards Mars. That is because the rotational period of Mars is greater than the orbital period of Phobos, which makes the relative position of Mars's tidal bulges behind that of Phobos. In about 40 millions years Phobos's inward spiral will end with its own disintegration [6].

As Triton orbits Neptune in retrograde motion, its tidally locked rotation leads to the same destiny as Phobos, making Triton slowly spiral inward until reaching Neptune's Roche limit in approximately 100 million years [6].

d) Tidal heating

As all the four Galilean satellites are relatively large, each orbit has been significantly affected by the action of the other three. In particular, the orbital periods of Io, Europa and Ganymede -the three nearer Jupiter- maintain a nearly perfect 1:2:4 ratio, which implies that the reciprocal gravitational tugs have finally become rhythmical, leading to synchronized orbits.

This orbital resonance forces Io, the innermost of the four and therefore the most tidally distorted by the planet, into a slightly eccentric orbit ($e = 0.004$). As Io goes around Jupiter barely varying its distance, also the strength of tidal forces inside the satellite barely change. But these continually varying tidal stresses make Io alternately squeeze and flex, generating so much heat through friction that much of its interior remains partially molten. This internal tidal heating makes Io the most volcanic body known in the whole universe [7].

Europa, the second of the Galilean satellites, is also forced to deform its orbits into an ellipse ($e = 0.01$) due to the orbital resonance exerted by Io and Ganymede. Therefore the effect of internal tidal heating is also generated, but with less intensity compared to Io, as the distance from Jupiter is greater. Nevertheless, this internal heat makes Europa the smoothest body in the Solar System, as its no-mountains-surface is been quietly but constantly reshaped through a water-and-ice version of plate tectonics [6].

Also Ganymede shows evidence of ancient tidal heating, as superficial founded furrows and grooves suggest once geologic activity, ceased a long time ago as its orbit became nearly circular ($e = 0.001$) [6].

Saturn's Enceladus and Dione are in an almost exactly 1:2 ratio of orbital periods, making the inner Enceladus to be caught in a rhythmical tug-of-war between Saturn and Dione. The resulting tidal heating is the most probable explanation for the geologic activity founded in Enceladus [6].

There also seems to be evidence of tidal heating in Uranus' Ariel, Titania, Miranda and Neptune's Triton [6].

e) Ring systems

All Jovian planets have rings obeying very strict geometrical laws: orbits must be almost perfectly circular, lie precisely in the equatorial plane, and be extremely flat.

The rings are formed by icy particles, ranging in size from about 1 cm to pieces 5 m across. Their orbital speeds across the rings must verify Kepler's third law, so the nearer particles to the planet move around more rapidly than the further ones.

As all ring systems lie within the Roche limits, probably their particles came from either being prevented from accreting into larger objects or being tidally disrupted from bodies approaching the planet.

Rings are dynamic, as particles are constantly being disturbed. That makes them short-lived in astronomical terms [8].

f) Overall loss of mechanical energy due to tidal forces

All the previously seen consequences of non-steady tidal forces imply friction. The total decrease in the mechanical energy of the Solar System due to tidal friction during its whole history has been estimated at 1.7×10^{33} kg m^2 s^{-2} (0.85% of all its present integral energy), having been transformed into heat [9].

Conclusion

Any celestial body placed in the gravitational field of another will experience tidal forces that tend to distort it, just because the attraction is stronger on one side than the other. Reciprocally, the generated distortion will affect the orbital parameters of the first object until a mutual stable condition could be achieved.

Therefore tidal forces not only have been reshaping the Solar System orbits, determining now and future presence, location and orbital parameters of satellite system around planets, but accounts also for having induced special and particular characteristics such as tidal heating, ring systems and even global climate changes.

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