Measuring the Solar Rotational Period

Abstract

The aim of this project is to measure the solar rotational period at the latitude of the sunspots and to observe them noticing their daily variation.

In first place, it will be discuss the necessity of using appropriate safe equipment to observe the Sun. Then, a summary of the sunspots major characteristics will be described, as well as an observed detailed development as a function of time. Next, the three methods here applied for measuring the day to day position of individual sunspots will be presented, followed by the corresponding recorded field data. At last, the solar rotational period will be calculated, being analyzed the accuracy of the three applied methods and compared its final results to the published data.

1) Observing the Sun

Immediate and permanent eye damage takes place from directly observing the Sun with inadequate protection. As the eye transmit most of the radiation between 3,800 and 14,000 angstroms -that is, the full visible light plus some near infrared radiation- to the light-sensitive retina, any incoming intense light source that surpass a safety threshold, which the Sun certainly exceeds, will result in an irreversible retinal burn [1].

Therefore, by no means any solar observation proceeding could even risk the slightly chance that direct sunlight could reach the unprotected observer's eyes.

There are just *only two different ways to safely observing the Sun*. One alternative is the "classic" way by projecting its image onto a screen, and the second option is by incorporating a proper solar filter at the "front end" of the telescope, where it will keep all but a tiny fraction of the Sun's light and heat out of the instrument. Any other method, like adding solar filters at the eyepiece, *certainly are not safe at all*.

Having been the solar filter alternative the only one used through all this project, this technique will be the described in detail from now on.

In order to assure adequate eye protection, the solar filter must provide a minimum filtering action, that is, the light transmittance should be limited up to maximum level of the ratio between the damage threshold and the solar spectral irradiance. For an extra margin of safety, the allowable transmittance is then set at about 1 percent of this ratio, thus reaching a transmittance of about 0.0032 percent [1], corresponding to the welder's filter shade # 12.

Several commercial solar filters can be obtained nowadays in the market that properly assure those safety requirements. Basically the standard filtering elements of all of them are made of a special metallized layer placed over a supporting surface like glass or polyester film, both types achieving "excellent performances" [2]. The metallized layer must be completely homogenous and cover 100% of its supporting base, becoming dangerous any pinhole or scratch, no matter how tiny it could be, as *otherwise direct sunrays would transit without being properly filtered.*

In particular, the sunscreen used in this project is showed in ***figure 1***. It is a 8" SCT *visual solar filter* made by *Kendrick Astro Instruments* (item # 6013) using a new metal film type, just developed in 1999: the "*Baader AstroSolar film*". Among excellent optical quality properties ("high contrast, scatter-free diffraction-limited image" [3]), this film assures superlative safety conditions as "it is coated on both sides, so that the chance of two pinholes overlapping each other is extremely faint" [4]. "The coating cannot easily be rubbed off, and in that respect this is actually safer than coated glass filters" [3]. The filter is factory installed into a lightweight circular metal cell, furnished with three nylon set screws to securely hold the filter at the front end of the telescope, preventing any possibility of careless displacement.

Figure 1
The visual filter Kendrick # 6013

Figure 2
The basic equipment used in this project

Figure 2 shows the basic equipment used through all the field work. The usual viewfinder had to be removed due to safety reasons, so that for each observational session the finding of the Sun's image have been achieved by means of minimizing the shadow projection of the telescope tube. Unless for high solar altitudes -around noon- when the cast tube shadow is been intercepted by the telescope, the daily solar disk localization was done fairly quickly.

2) Sunspots

The sunspots are regions of extremely strong magnetic field actually found on the Sun's photosphere, that is, inside the thin first layer of its atmosphere from which the solar visible light is emitted. Hence *the sunspots seem to be placed at the solar surface*, becoming the most obvious observational feature of what is called *solar activity*. Sunspots are a *dynamic phenomena*, appearing in constant evolution process of birth, reshaping and death, lasting from a minimum of few hours to a maximum of more than a month [5], at the same time that *may slightly drift their relative location over the surface* without any defined rule.

Each sunspot has a dark central core called the *umbra*, where the magnetic flux loop emerges vertically from below, being surrounded by a small lighter dark area called the *penumbra*, where the magnetic field spreads outward. The observed darkness with respect to the brilliance of the photosphere is due to *relative lower temperatures*: while the average photospheric temperature is about 5,800 K, penumbra temperatures typically range around 5,000 K and umbra temperatures around 4,000 K.

It appears to be a lower limit on the size of the spots of approximately 500 km [6] and also about their location: *they seldom are found at latitudes further than 35° from the solar equator*. Most spots do not occur singly but tend to appear in groups. *The formation, development and decay of large sunspot group may occupy several weeks*, but mostly of them are small, having lifetimes less than a couple of weeks due to being rapidly heated by radiation from the surroundings and destroyed [6].

Despite each sunspot gradually varies in size and also potentially could slightly change its location, *it is possible to determine the Sun's rotation rate by tracking them as they daily moved across the solar disk*. As the Sun does not rotate as a rigid solid, having different rotation rates at different latitudes -that is, *differential rotation*-, to accomplish that goal (at least around ± 30° from the equator) it is necessary to track several sunspots at different latitudes, the longer the observed period the more accurate the obtained final results.

3) Observing the sunspots

Any serious astronomical field work implies *the precise determination of the observed feature localization*. This means that any observed sunspot must be associated to its actual position, that is, its corresponding current *heliographic coordinates*.

"Heliographic" means measured with reference to the Sun's surface. The heliographic coordinates system is like the latitude-longitude system used on Earth. The heliographic latitude, usually referred as *B*, varies from 0 degrees at the Sun's equator up to ± 90 degrees at each solar pole. The heliographic longitude, *L*, is measured from a specific rotating "prime" meridian *in the same direction of rotation*, varying from 0 to 360 degrees. That specific prime meridian has been selected as the meridian that were at the center of the solar disk on November 9, 1853, 0:00 UT and *is considered to rotate at the uniform rate of 27.2753 days, as seen from the Earth*. That also defines the *Carrington Rotation*

Number, being the exact sequential number of the prime meridian rotations that had happened since that date.

Often it is far more convenient to know the spots' heliographic longitude *referred to the current solar disk*. This relative longitude corresponds to the "absolute" longitude L minus *the current longitude of the solar disk center*, usually called L_O. That makes that *the relative heliographic longitude* (L-L_O) is positive for locations at the west side of the solar disk and negatives at the east side, varying from +90° at the receding limb down to –90° at the approaching limb.

Because *the Earth doesn't orbit exactly around the Sun's equator*, as seen from the Earth through the year the center of the solar disk sinusoidally tips up and down up to 7.25 degrees, and also sinusoidally tilts eastward or westward up to 26.29 degrees. The tipped angle for a specific moment is called *Bo* and represents the *heliographic latitude of the current solar disk center.* The tilted angle for a specific time is called *P* and represents the *inclination of the current solar north direction*, being the *positive values towards the east side*.

Whatever sunspot image recording method can be used (photograph or sketch), besides the date and the time it is imperative to incorporate the exact position of the corresponding east-west Earth's direction at that moment, *otherwise the spots heliocoordinates could not be found*. The simplest way to find out this line is *by turning off the telescope astronomical drive*, so the solar image will immediately drift in that direction. With the only exception of the Carrington's method, the final accuracy of the measured spots heliocoordinates mostly will depend on the correctness of the recorded *EW* direction.

The daily number of observed sunspots can greatly vary in short periods like less than week, but the overall observed number considered over large periods (*i.e*. 3 months) changes far more smoothly and defines what is called the *sunspot cycle*, as the overall number seems to repeat maximums and minimums after about the same time lapse. *The sunspot cycle returns to a minimum after approximately 11 years*, having reached the last maximum two years ago. As the current solar cycle is about half way from the passed maximum (second half of 2000) and the next coming minimum (expected to 2007), this sunspot project has been done under favorable conditions: not too much spots that would complicate the identification and recording of its daily position, not too less spots that would imply scarce few data.

As an example of the daily variation in *number*, in *umbra and penumbra size*, in *shape* and in *the relative location* of a particular cluster of sunspots, ***figures 3 to 10*** show eight sketches corresponding to eight consecutive observations done from October 11, 2002 up to October 18, 2002. All of the sketches have been drawn at the same scale. The relative heliographic longitude of the center of each sketch along the sequence varies from about 45° east (for the first one) to about 45° west (for the last one), that is, an overall region of 90° centered at the central part of the solar disk (and thus *minimizing the usual visual deformation normally seen at larger longitudes*).

Among other little spots, three larger ones can noticeable be seen and they are identified as follows: the great spot that always appears at the left is *Spot N° 23*, the great one that always appears at the top is *Spot N° 24*, and the third one is *Spot N° 26*.

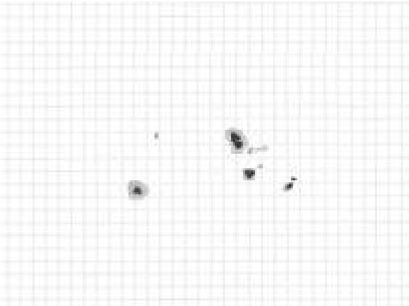


Figure 3: 11 Oct 2002, 19:19 UT
23 (N11 E36), 24 (N16 E46), 26 (N13 E42)

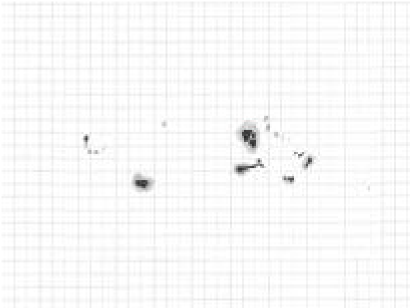


Figure 4: 12 Oct 2002, 12:45 UT
23 (N11 E27), 24 (N16 E36), 26 (N13 E35)

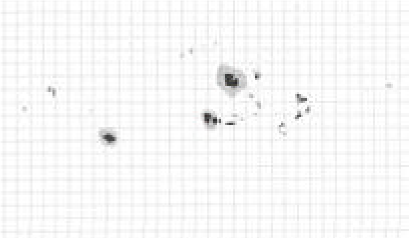


Figure 5: 13 Oct 2002, 14:25 UT
23 (N10 E13), 24 (N15 E23), 26 (N12 E20)

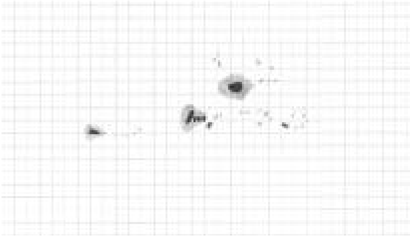


Figure 6: 14 Oct 2002, 15:38 UT
23 (N11 W01), 24 (N15 E09), 26 (N12 E06)

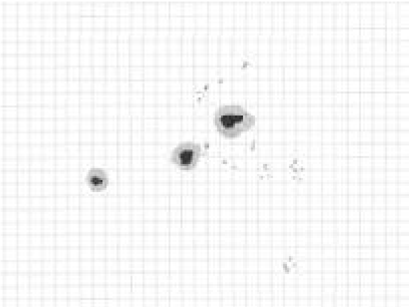


Figure 7: 15 Oct 2002, 19:29 UT
23 (N12 W16), 24 (N16 W06), 26 (N13 W09)

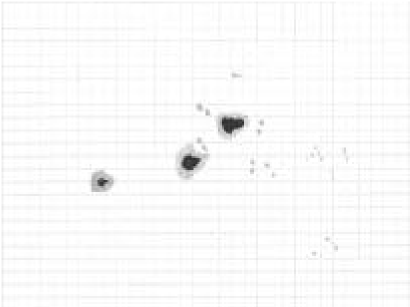


Figure 8: 16 Oct 2002, 18:44 UT
23 (N12 W28), 24 (N16 W18), 26 (N13 W21)

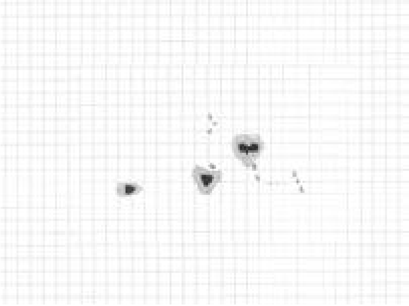


Figure 9: 17 Oct 2002, 19:11 UT
23 (N12 W42), 24 (N17 W32), 26 (N13 W35)

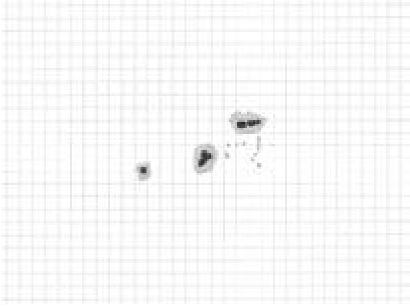


Figure 10: 18 Oct 2002, 18:53 UT
23 (N12 W55), 24 (N17 W44), 26 (N13 W47)

The detailed development of the big sunspots *N° 23, N° 24, N° 26* and a lot of little ones seen around during those eight days has been summarized in the following table describing the major observed changes:

Changes	***Spot N° 23***	***Spot N° 24***	***Spot N° 26***	***Overall comments***
from day 1 to day 2	Slightly changed in shape	Minimum changes	Noticeable size rise and moved to the left (west) side	Little spots appeared around; the spots at the right side has noticeable grown up
from day 2 to day 3	Slightly changed in shape	Slightly changed in shape	Slightly changed in shape and moved to the left (west) side	More little spots appeared around; the spots at the right side has notoriously grown up
from day 3 to day 4	Minimum changes	Slightly changed in shape	Changed in shape and moved to the left (west) side	More little spots appeared around; spots at the right side has almost lost their tiny umbra
from day 4 to day 5	Slightly changed in shape	Noticeable rise in the umbra size	Noticeable rise in the umbra size	Little spots appeared at the top (north) and bottom (south); maximum number of spots
from day 5 to day 6	No noticeable changes	Slightly changed in shape	Slightly changed in shape	The little spots of the left and bottom side has been notoriously scattered
from day 6 to day 7	No noticeable changes	Noticeable umbra reduction	Noticeable umbra reduction	Reduction of the overall number of little spots
from day 7 to day 8	Slightly changed in shape	Slightly changed in shape	Slightly changed in shape	New little spots has appeared and others has disappeared

4) Description of the applied methods for obtaining the sunspots coordinates

Once each solar image has been properly recorded at field (each observation must include the date, time, *EW* line, weather conditions, observation's place and Sun's altitude) the next task is *to measure the coordinates of each sunspot*. There are several methods to accomplish that objective: some ways are more straight and therefore less precise, others are quite the opposite.

The prepared graticule method

The simplest procedure is to apply a *specially prepared grid* to overlay the recorded full disk solar image (drawing or photograph) [7], as *it does not require any further computation*.

One type of prepared grids are the *Stonyhurst disks*, just being heliographic grids marked every 10° in latitude and 10° in longitude. There are eight different Stonyhurst disks, one for each specific $\pm B_O$ value, as the same disk can be used for the negative B_O value just by reversing the positive one.

The proceeding here described has been applied to the solar image obtained on September 20, 2002, 19:05 UT (B_O = + 7.10°, P = + 24.92°).

Figure 11
The solar image of 20 SEP 02 19:05 UT showing the EW line

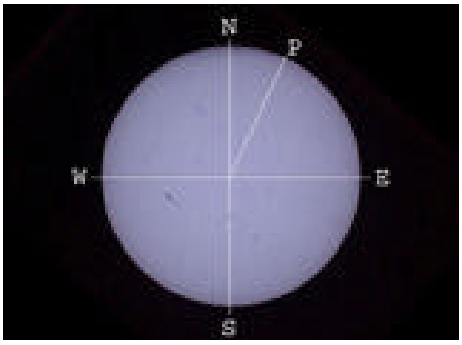


Figure 12
The true solar north has been drawn according to P = +24.9°

Once that the solar image has properly been associated with its corresponding *EW* line, as shown in ***figure 11***, then the north-south solar axis is drawn over the image, just by taking the right *P* angle from the north axis (in this case 24.92° to the east side, as ***figure 12*** shows).

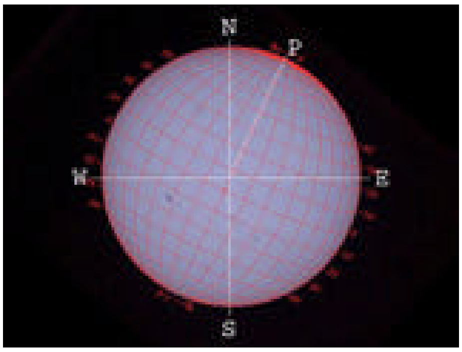


Figure 13
The Stonyhurst disk B_O = 7° has been overlaid over the P line

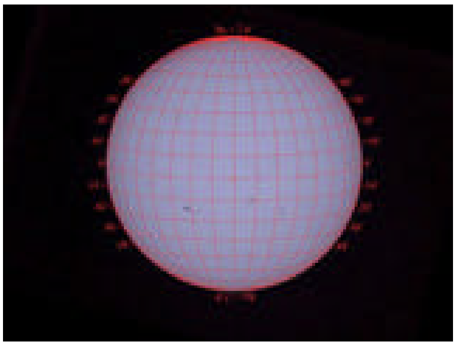


Figure 14
Over the final grid the true coordinates are easily find

Having the corresponding B_O Stonyhurst disk been transformed in size in order that it achieves the same diameter as the image, it must be rotated so that its vertical axis overlaps the *P* axis, as it is shown in ***figure 13***. Lastly, *each spot coordinates can directly be measured over the heliographic grid*, as appears in ***figure 14***.

The graphical method

This procedure has a first part where *some measurements must be made over the solar image* and a final part of *numerical computation* [8].

Figure 15 depicts a typical solar image with some spots and the corresponding *EW* line. The first step is to draw a line from the center of the disk, through the specific sunspot, and to the edge. On ***figure 16*** this line is marked *X*.

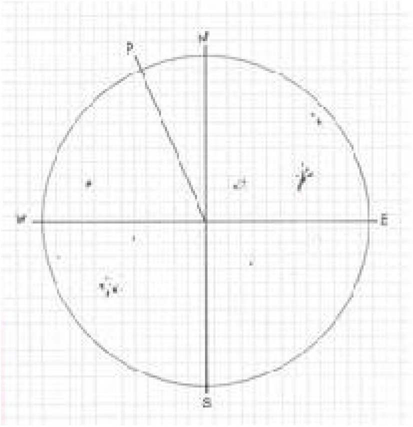


Figure 15
A solar image with many spots

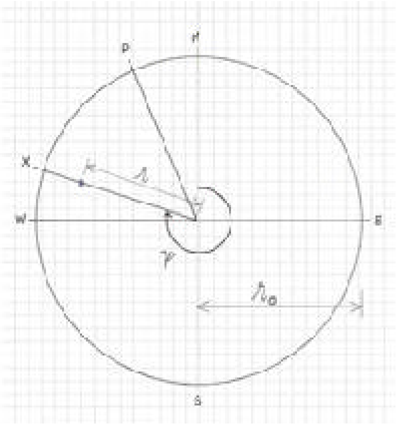


Figure 16
Measuring a specific sunspot

There are three measurements that must be done over that image: 1) the length r_o of the radius of the Sun's image; 2) the length r from the center of the disk to the sunspot; 3) the angle y between the north point of the disk to the sunspot, *measured towards the east side*.

Having measured those values and obtained the current solar ephemeris (B_O, L_O, P and *the apparent solar disk diameter*), the computation part begins finding the angle subtended from the Earth between the solar disk center and that sunspot, R, from

$$R = S \, {}^{r}\!/_{r_o} \qquad \textbf{(1)}$$

where S is the *current apparent angular radius of the Sun*.

Then it is necessary to calculate the angle r between the direction of the sunspot and the direction of the Earth, as seen from the center of the Sun. It becomes

$$\sin(R + r) = {}^{r}\!/_{r_o} \qquad \textbf{(2)}$$

Finally, the heliographic coordinates (latitude B and longitude L) of the sunspot can be obtained from the following formulae

$$\sin B = \sin B_O \cos r + \cos B_O \sin r \cos(P - y) \qquad \textbf{(3)}$$

$$\sin(L - L_O) = \sin r \sin(P - y) / \cos B \qquad \textbf{(4)}$$

The Carrington's method

This is the one of the most accurate proceedings. Unlike the two former described methods, *it does not required the use of a solar image*. The only particular needed elements are a cross-wire reticle eyepiece and a split-time stopwatch [9].

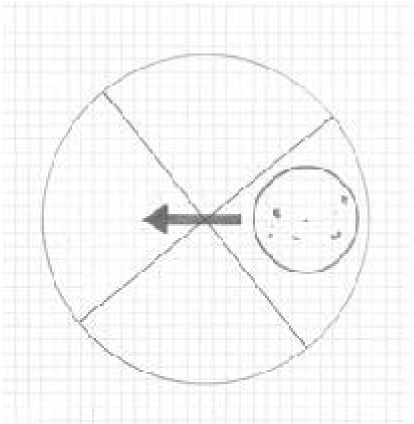


Figure 17
The Sun as seen throuhg the reticle eyepiece

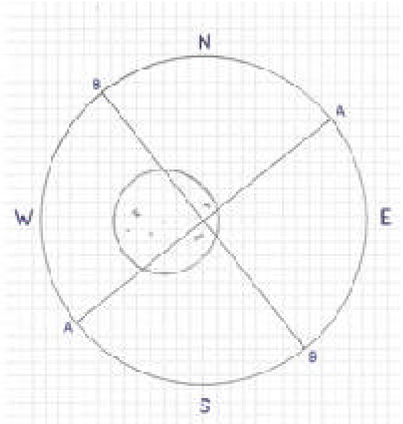


Figure 18
The specific cross-wire A and B axes

As shown in ***figure 17***, the reticule eyepiece field must be at least two solar diameters. The telescope should be use without any automatic drive, letting the solar image drift through the cross-wires from the east side to the west side.

At the beginning, the wires should be set about 45° to the direction of the drifting, *the precise angle being unimportant*, provided the preceding and following limbs of the Sun and the spot to be measured transit each wire at different times within the field of view.

The coordinates B and L of a sunspot may be derived from the timing measurements of six wire traversing times as the solar image goes across the field: the disk both first (preceding) and last (following) contacts and the sunspot transit. Therefore, it must be measured three transits through the A axis (the one that runs in the *NE* to *SW* direction) respectively A_p, A_f and a, and three more through the B axis (from *NW* to *SE*), B_p, B_f and b, as ***figure 18*** depicts.

The following computation are required. Firstly it is necessary to obtain the time of the transit of the solar disk through each axis:

$$A_C = (A_p + A_f)/2 \qquad \textbf{(5)}$$

$$B_C = (B_p + B_f)/2 \qquad \textbf{(6)}$$

The next step is to deduce the true position angle, e , of the *NE-SW* arm of the cross-wire:

$$\tan e = (A_f - A_p)/(B_f - B_p) \qquad \textbf{(7)}$$

A further angle, the position angle of the spot as measured from the *A* axis, a , is required:

$$\tan a_1 = \frac{a - A_C}{b - B_C} \frac{1}{\tan e} \qquad \textbf{(8)}$$

In order to convert a_1 to a it is necessary to apply a correction for the quadrant in which the position angle of the spot is to be found, according to:

***a* - *A*$_C$**	***b* - *B*$_C$**	**Value of a**
positive	positive	a_1
positive	negative	a_1 + 180°
negative	negative	a_1 + 180°
negative	positive	a_1 + 360°

The angle q subtended at the center of the disk by the line joining the spot and the north end of the solar axis is given by

$$q = a + (e + i - P) \qquad \textbf{(9)}$$

where *i* is the current inclination of the Sun's path to the parallel of declination passing through the disk center (a small value that goes from 0° to ± 0.06°) and *P* is the current position angle of the north end of the solar axis.

The true angular distance from the center of the Sun's disk, r , can be obtained from:

$$r/R = (2/\cos a)(b - B_C)/(B_f - B_p) \qquad \textbf{(10)}$$

$$r = r_1 - r_2 = Arc\sin(r/R) - 0.267(r/R) \qquad \textbf{(11)}$$

Finally, the heliocoordinates *B* and *L* can be calculated from:

$$\sin B = \cos q \cos B_O \sin r + \sin B_O \cos r \qquad \textbf{(12)}$$
$$\sin(L_O - L) = (\sin q \sin r)/\cos B \qquad \textbf{(13)}$$

The Carrington's method is "capable of yielding results comparable in accuracy with the photographic, provided the instrument is stable, the observer practiced and a sufficient number of transits secured to eliminate accidental errors in noting the times of contact" [9].

The final formulae of the Carrington's method are basically the same as the graphical method, with two minor differences: 1) the angle q corresponds to the "graphical" angle (P - y) and 2) the longitudes formula have opposite signs, as the Carrington's is referred to ($L_O - L$) while the "graphical" is ($L - L_O$).

5) The obtained data

The prepared graticule method

The solar field work has been done from September 20, 2002, until November 10, 2002, that is during more than 7 weeks, achieving 43 daily observations. All of them were carried out from *Salto, Uruguay* (S 31° 23', W 57° 59') except those of October 19 and 22, which were made from *Montevideo, Uruguay* (S 34° 52', W 56° 03').

It has been used a 8" Schmidt-Cassegrain telescope *Meade LX-90* working at 50X (40 mm eyepiece *Meade Super Plossl Series 4000*), with a digital camera *Olympus C-2020 Zoom*. All the digital photos has been taken from the eyepiece image by means of the *afocal method*. Hence, all the obtained solar digital images are *mirror reversed* (whenever north is up, east is at the right side of the image and vice versa).

In *Appendix I* are shown each one of the processed digital images with their corresponding Stonyhurst disks properly overlapped. The disks have been obtained from the internet [10] and all the software process has been done in *Adobe Photoshop 6.0* software. In *Appendix II* is the excel page with all the daily measured sunspots coordinates. The spots has been identified by an arbitrary correlative number and also by their corresponding "*NOAA Sunspot Number*", that is, the "official" sunspot number tabulated by the *US National Oceanographic and Atmospheric Administration* in its solar daily report [11].

Despite technically being considered belonging to the same group (as their separation were less than 5 degrees [12]), large noticeable separated umbra areas has been identified with different numbers and by assigning different letters to the same *NOAA number*.

The overall results have been as shown in the table of the following page.

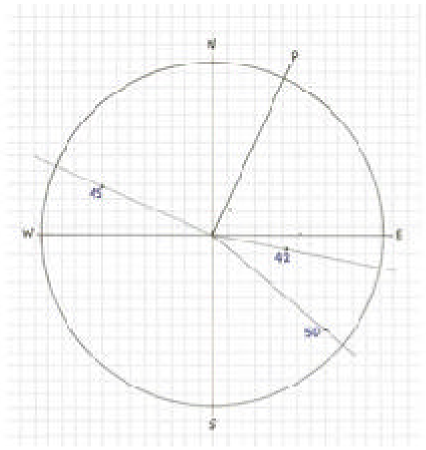


Figure 19
The main sunspots of 02 NOV 2002

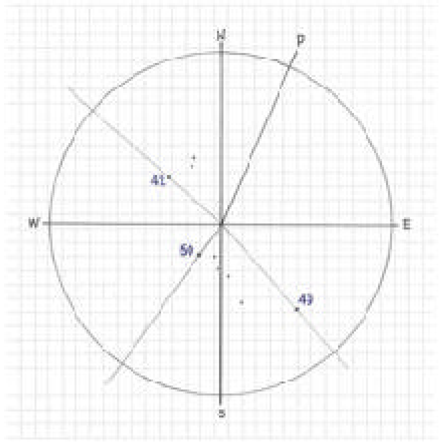


Figure 20
The main sunspots of 06 NOV 2002

Spot		***Mean latitude (°)***	***Total*** ΔL ***(°)***	***Total elapsed hours***	$\Delta L / \Delta h$ ***(°/h)***
N°	**NOAA**				
1	115	-4	26	45.3	0.574
2	119	-14	55	97.1	0.566
3	123	-17	40	72.6	0.551
4	117	-9	80	144.6	0.553
5	121	-13	87	165.8	0.525
6	122	-17	79	144.6	0.546
7	127	-13	79	144.6	0.546
8	126	-22	40	74.0	0.541
9	132b	18	61	120.5	0.506
10	132a	19	48	93.2	0.515
11	130a	5	93	167.3	0.556
12	130b	7	38	68.7	0.553
13	134a	12	110	212.3	0.518
14	134b	14	88	167.7	0.525
15	134c	9	91	167.7	0.543
16	137a	-19	94	169.3	0.555
17	137b	-19	72	139.5	0.516
18	139	12	141	260.1	0.542
19	141	-7	31	52.8	0.587
20	140	-7	134	244.2	0.549
21	142	6	27	48.2	0.560
22	144	12	107	191.4	0.559
23	145	11	127	235.3	0.540
24	149a	16	126	235.3	0.535
25	146	-8	12	23.3	0.515
26	149b	13	107	189.0	0.566
27	148	-20	101	190.9	0.529
28	154a	-13	52	96.9	0.537
29	154b	-13	14	25.2	0.556
30	158	-9	122	214.5	0.569
31	159	-12	80	146.0	0.548
32	160a	-18	40	69.3	0.577
33	160b	-23	36	69.3	0.519
34	162a	25	155	287.5	0.539
35	162b	25	144	269.4	0.535
36	165	20	118	222.7	0.530
37	167	18	82	152.4	0.538
38	169	-19	44	80.3	0.548
39	171	10	83	152.4	0.545
40	175a	15	122	214.5	0.569
41	174	-25	38	72.1	0.527
42	176	10	155	285.4	0.543
43	175b	15	37	70.4	0.526
44	177a	17	126	237.0	0.532
45	178	2	57	93.3	0.611
46	179	3	14	25.3	0.553
47	180a	-11	116	213.3	0.544
48	182	-18	100	188.0	0.532
49	185	-12	78	143.3	0.544
50	180b	-10	67	118.7	0.564
51	188	11	44	76.6	0.574
52	190	-21	40	70.9	0.564
53	177b	19	39	70.9	0.550

The graphical method

The measured values obtained by applying equations **(1)** through **(4)** to the solar drawing of ***figure 19***, corresponding to November 2, 2002, 19:50 UT (B_O = 4.21°, P = 24.22°, D = 2S = 32’ 17.5’’) are:

Spot	r_o **(mm)**	***r*** **(mm)**	***R*** **(°)**	*r* **(°)**	*y* **(°)**	***B*** **(°)**	***L-L$_o$*** **(°)**
N° 45 (178)	61.8	43.5	0.1894	44.55	293.7	**2.6**	**44.6**
N° 50 (180b)	61.8	53.2	0.2317	59.18	130.0	**-11.3**	**-57.4**
N° 42 (176)	61.8	27.5	0.1197	26.30	100.8	**9.7**	**-25.9**

In the same way, the measured values obtained from the solar drawing of ***figure 20***, corresponding to November 6, 2002, 15:20 UT (B_O = 3.81°, P = 23.49° and D = 2S = 32’ 19.4’’) are:

Spot	R_o **(mm)**	***r*** **(mm)**	***R*** **(°)**	*r* **(°)**	*y* **(°)**	***B*** **(°)**	***L-L$_o$*** **(°)**
N° 42 (176)	61.8	24.8	0.1081	23.55	312.0	**10.8**	**22.7**
N° 49 (185)	61.8	41.0	0.1787	41.38	138.9	**-13.5**	**-37.9**
N° 50	61.8	13.5	0.0588	12.56	216.1	**-8.4**	**2.7**

(180b)							

The Carrington's method

In *Appendix III* there is a detailed photographic sequence of the application of this method, and in *Appendix IV* it is described the construction process of the crosswire reticle adaptation needed for making the timings.

The timing measurements obtained on November 2, 2002, were:

UT Time		***Disk first contact***	***Spot N° 45***	***Spot N° 40***	***Spot N° 42***	***Spot N° 44***	***Disk last contact***
19:13	*A axis*	10.39	47.80	109.92	151.24	148.43	216.76
	B axis	0.00	62.89	100.09	115.43	125.73	177.24
19:18	*A axis*	6.92	41.68	102.19	144.09	142.30	211.25
	B axis	0.00	62.58	104.52	113.68	125.08	177.92
19:24	*A axis*	12.02	48.60	109.70	149.64	147.39	213.69
	B axis	0.00	61.70	102.81	113.95	124.81	175.82
19:29	*A axis*	13.96	48.74	106.65	151.99	150.20	216.90
	B axis	0.00	63.56	104.08	114.92	124.86	179.04
19:35	*A axis*	0.00	24.63	82.54	117.38	119.98	166.39
	B axis	2.59	97.09	134.76	140.13	152.36	225.55

The corresponding solar ephemeris values (19:30 UT) were: B_O = 4.21°, L_O = 2.14°, i = -0.05° and P = 24.22°.

Example of the computation for the 19:35 UT measurements:

According to **(5)** and **(6)**

$$A_C = (0 + 166.39)/2 = 83.20 \text{ s} \qquad B_C = (2.59 + 225.55) / 2 = 114.07 \text{ s}$$

From **(7)**, $\tan e = \dfrac{166.39 - 0}{225.55 - 2.59} = 0.7463 \Rightarrow e = 36.73°$

For the *Spot N° 45 (178)*, the formula **(8)** results

$$\tan a_1 = \frac{24.63 - 83.20}{97.09 - 114.07}\,\frac{1}{0.7463} = 4.6219 \Rightarrow a_1 = 77.79°$$

and according to the a converting table, in that case a becomes a_1 + 180°, so a = 257.79°

From the ephemeris table for that time and date, i = 0.05° and P = 24.22°, so the equation **(9)** becomes

q = 257.79 + (36.73 – 0.05 – 24.22) = 270.25°

Hence **(10)** becomes $r/R = \dfrac{(2/\cos 257.99)(97.09 - 114.07)}{225.55 - 2059} = 0.7202$

Making $r_1 = 46.07°$ and $r_2 = 0.19$ °, so that from **(11)** r = 45.88°

Finally, from **(12)** and **(13)**

sin B = cos 270.25° cos 4.21° sin 46.07° + sin 4.21° cos 45.88° ⇒ B = 3.11°
sin ($L_O – L$) = sin 270.25° sin 4.21° / cos 3.11° ⇒ $L_O – L$ = - 45.97°

In the same way, having applied the same computation process for each other timing values it has been achieved the following angular results:

Time		***Spot N° 45 (178)***					***Spot N° 40 (175a)***				
	e	*a*	*q*	*r*	***B***	***L_O-L***	*a*	*q*	*r*	***B***	***L_O-L***
19:13	49.34	245.51	270.58	44.27	**3.42**	**-44.37**	344.67	369.74	7.67	**11.77**	**1.32**
19:18	48.95	245.80	270.48	46.15	**3.26**	**-46.24**	338.89	363.57	10.76	**14.95**	**0.69**
19:24	48.92	244.93	269.58	44.53	**2.71**	**-44.59**	349.52	374.17	9.88	**13.78**	**2.48**
19:29	48.58	246.19	270.50	45.73	**3.30**	**-45.83**	331.99	356.30	10.57	**14.76**	**-0.70**
19:35	36.73	257.79	270.25	45.88	**3.11**	**-45.97**	357.55	370.01	10.66	**14.70**	**1.90**
Mean					**3.16**	**-45.40**				**13.99**	**1.14**
			Abs.max.differ:		***0.45***	***1.03***		***Abs.max.differ:***		***2.22***	***1.84***

Time		***Spot N° 42 (176)***					***Spot N° 44 (177a)***				
	e	*a*	*q*	*r*	***B***	***L_O-L***	*a*	*q*	*r*	***B***	***L_O-L***
19:13	49.34	50.34	75.41	28.16	**10.56**	**27.68**	38.89	63.96	32.41	**17.25**	**30.28**
19:18	48.95	50.96	75.64	26.06	**10.06**	**25.61**	38.68	63.36	31.20	**17.12**	**28.98**
19:24	48.92	50.92	75.57	27.90	**10.44**	**27.44**	39.21	63.86	32.66	**17.39**	**30.51**
19:29	48.58	51.78	76.09	27.18	**10.07**	**26.76**	40.96	65.27	31.38	**16.26**	**29.52**
19:35	36.73	60.36	72.82	28.08	**11.74**	**27.34**	52.15	64.61	33.89	**17.42**	**31.87**
Mea					**10.57**	**26.97**				**17.09**	**30.23**

n											
			Abs.max.differ:		***1.17***	***1.36***		***Abs.max.differ:***		***1.16***	***1.64***

The timing measurements obtained on November 6, 2002, were:

UT Time		***Disk first contact***	***Spot N° 42***	***Spot N° 50***	***Disk last contact***
14:35	*A axis*	3.50	58.53	102.67	194.72
	B axis	0.00	90.16	70.79	188.10
14:39	*A axis*	0.00	54.33	94.23	184.91
	B axis	1.46	98.19	76.11	197.83
14:43	*A axis*	5.64	58.51	97.41	185.22
	B axis	0.00	101.09	78.16	204.39
14:54	*A axis*	19.76	83.63	136.61	231.41
	B axis	0.00	77.76	65.49	170.79
14:59	*A axis*	0.00	62.57	109.11	205.95
	B axis	7.41	90.94	75.32	185.91

The corresponding solar ephemeris values (15:00 UT) were: B_O = 3.81°, L_O = 311.87°, i = -0.05° and P = 23.49°. Applying the proper computation process by fomulae **(5)** to **(13)**:

Time		***Spot N° 42 (176)***					***Spot N° 50 (180b)***				
	e	*a*	*q*	*r*	***B***	***L_O-L***	*a*	*q*	*r*	***B***	***L_O-L***
14:35	45.47	263.91	285.84	22.85	**9.61**	**-22.27**	163.38	185.31	14.89	**-11.02**	**-1.39**
14:39	43.28	267.94	287.68	24.33	**10.68**	**-23.54**	175.43	195.17	13.86	**-9.56**	**-3.64**
14:43	41.30	268.49	286.25	24.23	**10.09**	**-23.59**	174.64	192.40	13.61	**-9.48**	**-2.94**
14:54	51.10	257.29	284.85	23.88	**9.46**	**-23.37**	155.93	183.49	14.72	**-10.88**	**-0.91**
14:59	49.08	260.72	286.26	23.31	**9.88**	**-22.68**	166.02	191.16	14.19	**-10.11**	**-2.76**
mean					**9.94**	**-23.09**				**-10.21**	**-2.33**
			Abs.max.differ:		***0.74***	***0.82***		***Abs.max.differ:***		***0.81***	***1.42***

From the obtained six spots heliocoordinates by applying this method, all of them by prorating five different time measurements, the major absolute individual difference of the 30 results has been a mere 2.22°. This exemplifies the precision of the Carrington's method.

6) The acquisition of related parameters

The daily Sun's ephemeris (*Bo, Lo, i, P*, the *apparent diameter* and the *Carrington rotation number*) for the time period covered by this project has been obtained from the internet [13] as follows:

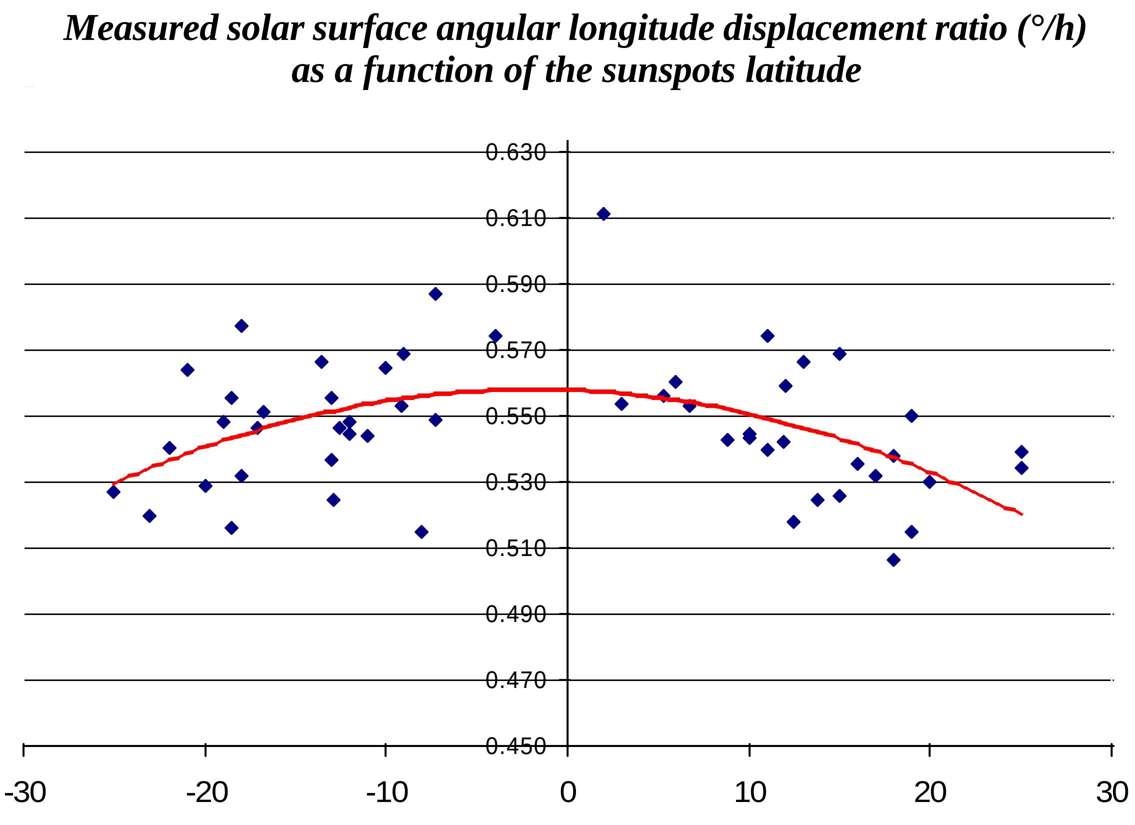
	Date		**B_o**	**L_o**	***i***	***P***	**Apparent Solar Disk Diameter**		**Carrington**
year	***month***	***day***	***(deg)***	***(deg)***	***(deg)***	***(deg)***	***(')***	***(")***	***Rotation***
2002	Sep	19	7.14	233.33	-0.06	24.66	31	53.5	1994
2002	Sep	20	7.12	220.13	-0.06	24.81	31	54.0	1994
2002	Sep	21	7.09	206.93	-0.06	24.95	31	54.6	1994
2002	Sep	22	7.06	193.73	-0.06	25.08	31	55.1	1994
2002	Sep	23	7.04	180.53	-0.06	25.20	31	55.6	1994
2002	Sep	24	7.00	167.33	-0.06	25.32	31	56.2	1994
2002	Sep	25	6.97	154.14	-0.06	25.43	31	56.7	1994
2002	Sep	26	6.94	140.94	-0.06	25.54	31	57.2	1994
2002	Sep	27	6.90	127.74	-0.06	25.64	31	57.8	1994
2002	Sep	28	6.86	114.54	-0.06	25.73	31	58.3	1994
2002	Sep	29	6.82	101.35	-0.06	25.82	31	58.8	1994
2002	Sep	30	6.77	88.15	-0.06	25.89	31	59.3	1994
2002	Oct	1	6.73	74.95	-0.06	25.97	31	59.9	1994
2002	Oct	2	6.68	61.76	-0.06	26.03	32	0.4	1994
2002	Oct	3	6.63	48.56	-0.06	26.09	32	1.0	1994
2002	Oct	4	6.58	35.37	-0.06	26.14	32	1.5	1994
2002	Oct	5	6.53	22.17	-0.06	26.18	32	2.0	1994
2002	Oct	6	6.47	8.98	-0.06	26.22	32	2.6	1994
2002	Oct	7	6.41	355.79	-0.06	26.25	32	3.1	1995
2002	Oct	8	6.35	342.59	-0.06	26.27	32	3.7	1995
2002	Oct	9	6.29	329.40	-0.06	26.28	32	4.3	1995
2002	Oct	10	6.23	316.21	-0.06	26.29	32	4.8	1995
2002	Oct	11	6.16	303.02	-0.06	26.29	32	5.4	1995
2002	Oct	12	6.10	289.83	-0.06	26.28	32	5.9	1995
2002	Oct	13	6.03	276.63	-0.06	26.27	32	6.5	1995
2002	Oct	14	5.96	263.44	-0.05	26.25	32	7.1	1995
2002	Oct	15	5.89	250.25	-0.05	26.22	32	7.6	1995
2002	Oct	16	5.81	237.06	-0.05	26.18	32	8.2	1995
2002	Oct	17	5.74	223.87	-0.05	26.14	32	8.7	1995
2002	Oct	18	5.66	210.68	-0.05	26.08	32	9.3	1995
2002	Oct	19	5.58	197.49	-0.05	26.02	32	9.8	1995
2002	Oct	20	5.50	184.30	-0.05	25.96	32	10.4	1995
2002	Oct	21	5.41	171.11	-0.05	25.88	32	10.9	1995
2002	Oct	22	5.33	157.92	-0.05	25.80	32	11.5	1995
2002	Oct	23	5.24	144.73	-0.05	25.71	32	12.0	1995
2002	Oct	24	5.16	131.54	-0.05	25.61	32	12.5	1995
2002	Oct	25	5.07	118.35	-0.05	25.50	32	13.1	1995
2002	Oct	26	4.98	105.16	-0.05	25.39	32	13.6	1995
2002	Oct	27	4.88	91.98	-0.05	25.27	32	14.1	1995
2002	Oct	28	4.79	78.79	-0.05	25.14	32	14.6	1995
2002	Oct	29	4.69	65.60	-0.05	25.00	32	15.1	1995
2002	Oct	30	4.60	52.41	-0.05	24.85	32	15.6	1995
2002	Oct	31	4.50	39.23	-0.05	24.70	32	16.1	1995
2002	Nov	1	4.40	26.04	-0.05	24.54	32	16.6	1995

2002	Nov	2	4.30	12.86	-0.05	24.37	32	17.1	1995
2002	Nov	3	4.19	359.67	-0.05	24.19	32	17.6	1996
2002	Nov	4	4.09	346.48	-0.05	24.01	32	18.1	1996
2002	Nov	5	3.98	333.30	-0.05	23.82	32	18.6	1996
2002	Nov	6	3.88	320.11	-0.05	23.62	32	19.1	1996
2002	Nov	7	3.77	306.93	-0.05	23.41	32	19.6	1996
2002	Nov	8	3.66	293.75	-0.04	23.19	32	20.0	1996
2002	Nov	9	3.55	280.56	-0.04	22.97	32	20.5	1996
2002	Nov	10	3.44	267.38	-0.04	22.74	32	21.0	1996
2002	Nov	11	3.33	254.19	-0.04	22.50	32	21.5	1996

As all the former data correspond to 00:00 UT daily times, the corresponding values for a given time has been obtained by prorating them among the data of the current and the following day.

7) Determination of the solar rotational period

From the daily observed sunspots data at the excel page of *Appendix II*, the following graph has been obtained plotting *each spot's latitude* versus *its hourly angular displacement ratio* (the measured overall angular longitude observed displacement divided by its corresponding total elapsed time). The tendency line has been selected from a *second grade polynomial* type, as the rotational rate should be symmetrical with respect to the latitude, reaching its maximum at the Sun's equator.



This graph shows a measured displacement angular rate of 0.56 °/h at the equator and about 0.53 °/h at ± 25°, which means a ***rotation rate of 26.8 and 28.3 days*** respectively. (As the "prime" meridian by definition rotates at 27.2753 days, at least those found results are among the expected values).

Those solar rotation periods are *synodic* values, that is, how long it takes the solar surface to make a full rotation *as seen from the Earth*. The *sidereal* rotation period, that is, *referred to the stars*, can be calculated from

$$P_{Sider} = \frac{1}{1/P_{Synod} + 1/P_{Earth}}$$

where P_{Sider} is the sidereal rotation period, P_{Synod} is the measured synodic rotation period and P_{Earth} is the Earth's sidereal rotation period (365.25 days). Hence, for the equator and the ± 25° latitude measured synodic rotational values it results that their respective sidereal periods are:

$$P_{Sider\,(0°)} = \frac{1}{1/26.8 + 1/365.25} = \mathbf{25.0\ days}$$

$$P_{Sider\,(25°)} = \frac{1}{1/28.3 + 1/365.25} = \mathbf{26.3\ days}$$

That is, ***the calculated solar sidereal rotation periode is 25.0 days at the equator and about 26.3 days at latitudes of ± 25°.***

8) Analysis of the obtained results

Two major analysis can be made from the obtained results: a) the potential accuracy in the determination of the sunspots coordinates referred to each applied method; b) the achieved accuracy of the solar rotation rate.

a) Comparison among the obtained heliographic coordinates by applying the three different methods to the same spot *at almost the same time*:

Measured at 02 NOV 2002	***Stonyhurst (19:50 UT)***	***Graphical (19: 40 UT)***	***Carrington's (19:20 UT)***	***Mean coordinates values***
Spot N° 42 (176)	N10 E26	N09.7 E25.9	N10.57 E26.97	***N10.1 E26.3***
Spot N° 45 (178)	N02 W45	N02.6 W44.6	N03.16 W45.40	***N02.6 W45.0***

Measured at 06 NOV 2002	***Stonyhurst (13:48 UT)***	***Graphical (15:19 UT)***	***Carrington's (14:50 UT)***	***Mean coordinates values***
Spot N° 42 (176)	N10 W22	N10.8 W22.7	N09.94 W23.09	***N10.2 W22.6***
Spot N° 50 (180b)	S09 W02	S08.4 W02.7	S10.21 W02.33	***N09.2 W02.3***

The results obtained by the three methods has been *almost identical*, as they reached to practically the same final values in the four cases, being the maximum absolute differences just 0.4° in longitude and 0.6° in latitude. The Stonyhurst disk method has an intrinsic accuracy of at least ± 1°, but it seems to be more easy and straight than the other two.

The spots *NOAA* locations (*given for a specific time once a day*) can be obtained for any time by prorating their respective values for two consecutive days. Here is an example:

02 NOV 2002	***00:30 UT***		***24:40 UT***		***Prorating for 19:40 UT***
Spot NOAA 176	N10 E37	*L = 335°*	N10 E24	*L = 335°*	***N10 E26.3***
Spot NOAA 178	N01 W30	*L = 42°*	N01 W45	*L = 44°*	***N01 W42.3***

The obtained coordinates from any of the three applied methods for the observed *Spot N° 42 (176)* on November 2, 2002, about 19:40 UT, are the same as the *NOAA* prorated location. However, for the *Spot N° 45 (178)* measured on the same day and time, the *NOAA* prorated location has a difference of almost 2° in latitude and nearly 3° in longitude, but it must be said that *this spot drifted 2° to the west on that day*, which at least partially accounts for the longitude discrepancy.

b) Comparison between the displacement ratios $\Delta L / \Delta h$ obtained through the three measurement methods for the *Sunspot N° 42 (176)* for a time period of about 4 days:

	$L-L_O$ measured 02 NOV 2002	***$L-L_O$ measured 06 NOV 2002***	***Longitude Displacement***	***Elapsed time in hours***	$\Delta L / \Delta h$
Stonyhurst	E26	W22	48°	91.7	0.5234
Graphical	E25.9	W22.7	48.6°	91.5	0.5311
Carrington's	E26.97	W23.09	50.06°	90.0	0.5562
mean value					***0.5369***

Hence, for this specific comparative case each calculated rotational rate has a difference with respect to the mean value of less than 3.6%. Also, in this case it is worth comparing that mean value with the calculated from the *NOAA* supplied coordinates for the *Sunspot N° 176* at November 02, 2002, 00:30 UT and at November 06, 2002, 00:40 UT, as the spot did not drift through the period (for both dates *L = 335°*).

	$L-L_O$ supplied 02 NOV 2002	***$L-L_O$ supplied 06 NOV 2002***	***Longitude Displacement***	***Elapsed time in hours***	$^{\Delta L}/_{\Delta h}$
NOAA	E37	W15	52°	96.2	***0.5405***

The difference between the calculated mean rotational rate (0.5369 °/h) and the obtained from the *NOAA* values (0.5405 °/h) is less than 1% for the example considered. Thus, by extrapolating this results it conservatively can be said that the overall accuracy of the found rotational periods in this field work are much better than 10%.

Finally, the published data for the solar sidereal rotation period vary from "*25 days at the equator and 27.5 days at latitude 30 degrees*" [5], "*25.4 days at the equator and 36 days near the poles*" [12], and "*about 25.6 days near the equator and 30.9 days at a latitude of 60 degrees*" [14], while the mean sidereal rotation period corresponding to the uniform rate of the Carrington rotation is 25.38 days. The here obtained values of 25 days at the equator and 26.3 days at latitudes of 25 degrees are well according within those values.

Conclusion

The sunspots are dynamic features that can be easily observed if the right safe equipment is used. In spite of their intrinsic drifting, following its day to day position it can be measured the solar differential rotation, the longer the study period the more accurate the obtained results.

The final results obtained from this short solar observational period -little more than 7 weeks with actual 43 daily observations- has achieved the satisfactory solar sidereal rotation periods of 25.0 days at the equator and 26.3 days at latitudes of 25 degrees, which are in full agreement with the current known values.

References

[1] "*Solar Filters Safety*" by B. Ralph Chou, Sky & Telescope, February 1998, page 37

[2] "*Solar Filters: Which Is Best?* " by Alan M. MacRobert, Sky & Telescope, July 1999, page 67

[3] "*Baader AstroSolar and Turbofilm Media*" by Company Seven, http://www.company7.com/astrophy/options/apasolar.html

[4] "*Solar Filters*" by Kendrick Astro Instruments, http://www.kendrick-ai.com/baader_tech.html

[5] "*Universe: Stars and Galaxies*" by Roger A. Freedman and William J. Kaufmann III, 6ed, page 406, R.R. Donnelley & Sons Company, 2002

[6] "*The Sun*", www.geocities.com/CapeCanaveral/Cockpit/9952/SUN1.doc

[7] “*Latitude/Longitude Grids*”, http://solar-center.stanford.edu/solar-images/latlong.html, and“*Using the Stonyhurst Discs*”, http://www.popastro.com/sections/solar/chap4.htm

[8] “*Advanced Amateur Astronomy*” by Gerald North, 2ed, pages 259 to 264, Cambridge University Press, 1998

[9] “*Carrington’s method of determining sunspot positions*” by E.T.H. Teague, The Journal of the British Astronomical Association, 106, 2, April 1996, pages 82 to 85

[10] “*Stonyhurst disks*”, http://www.meadows3.demon.co.uk/html/stonyhurst.html

[11] “*USAF/NOAA Solar Region Summary daily reports*”, http://www.sec.noaa.gov/ftpdir/forecasts/SRS/0920SRS.txt

[12] “*Solar Observing FAQ*”, http://www.roboticobservatory.com/jeff/faq/index.htm

[13] “*Solar ephemeris 2002*”, http://www.meadows3.demon.co.uk/data/heliocoords2002.txt

[14] “*Sacramento Peak: The Sun*”, http://www.sunspot.noao.edu/sunspot/pr/answerbook/sun.html

Appendix I

The processed digital images

Appendix II

The measured sunspot coordinates data base

Appendix III

Photographic sequence of the Carrington's method

Appendix IV

The construction of a reticle adaptation for a common eyepiece

As the Carrington's method requires a crosswire reticle eyepiece of about 50 mm of focus distance, that is not so easy to obtain, I had to manage to adapt my common 40 mm eyepiece.

Two practical problems had to be resolved: 1) the crosswire must be as perfect as possible, that is, very thin axes and at truly right angles; 2) the crosswire reticle location must accept fine tune position in order to be placed at the exact focal plane –otherwise no sharp axes could be seen.

The final solution was conceived after obtaining a spare disk reticle from a precision "pocket comparator/magnifier" from Mitutoyo (series # 183, disk 06, ref. #183-106).

Having checked that the focal plane of my common Plossl 40 mm eyepiece was "accessible", I make a precise measurement of all the required dimensions (sketch attached) for ordering the construction of a special aluminum cylinder piece that would carry the reticle disk inside the eyepiece. It also has to be threaded in order to be fastened inside the eyepiece at the same place where normal disk filters are held (about 18 mm further from the focal plane).

The photos show the disk reticle definitely placed inside the special cylinder piece (at right in first row), the eyepiece with its metal bracket cylinder separated and lined up with the "new" reticle piece (second row at left), and the full assembled reticle eyepiece (at right in second row).