Diameter and Depth of Lunar Craters

Abstract

The aim of this project is to find out the size and height of some features on the surface of the Moon from obtained observational field measures and the afterwards proper calculation in order to transform them into the corresponding physical final values.

First of all it will be briefly described two different proceedings for the acquisition of the observational field measures and the theoretical development of the formulae that allow the conversion into the final required results. It will also be discussed the implicit theoretical errors that each method presents.

Secondly, it will be described the actual applied measuring proceedings and the complete exposition of the field collected data, plus photos and sketches.

Then it will be presented all the lunar related parameters corresponding to the same moments as when the measures were obtained, basically taken from the internet.

After that, through the application of the proper formulae to each observational measure and the corresponding related parameters, it will be found out the required lunar features sizes and heights.

Later the obtained final values will be compared to the actual values, analyzing the possible reasons of the differences, and suggesting improvements and special cares in the experimental application of the performed proceedings.

Finally, an overall conclusion and the complete list of all the involved references will end this project report.

1) Theoretical considerations

a) Measuring lunar surface sizes

In order to measure the size of lunar features by telescopic observation from the Earth, it is necessary to firstly obtain some direct value related with its size and secondly apply the corresponding calculation that converts it into the desired actual length.

The more precise method for obtaining any lunar feature measurement from telescopic direct observation is by means of a calibrated micrometric eyepiece. However, in this project another two alternative techniques have been applied: the "*photographic method*" and the "*drift timing method*".

The "***photographic method***" is based on the measurement of the length from an image feature obtained by taking a digital photograph through the telescope and then applying the proper conversion to obtain the measured image length in pixels.

Once the digital photograph has been introduced in a PC, any length can be precisely measured in pixels by the aid of a photo processing software (Microsoft Photo Editor, Adobe Photoshop, etc). Those programs not only allow *enlarging as required* the feature image thus achieving more precision, but also determine *exactly* each selected pixel location (x_i, y_i).

Applying Pythagoras's theorem, the pixel length (m) between any two points P_1 (x_1, y_1) and P_2 (x_2, y_2) is

$$m = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \textbf{(1)}$$

That image size in pixels must be converted to a physical size in meters, multiplying by the *scale factor* (*SF*). To determine the *scale factor*, it is necessary to know how many meters correspond to one pixel. One way is by knowing the physical size of any feature that appears on the same photo (it could be even the diameter of the Moon), measuring how many pixels takes its feature image.

Due to the spherical surface of the Moon, any feature placed not exactly on its apparent center disc *is observed under some angle of inclination*. The further the observed feature from the currently apparent center of the Moon's disc, the greater the deformation of its image, inclusive being differently deformed in the north-south direction respect to the east-west direction, depending on the *actual location of the lunar feature*. If that is the case, the measured length in pixels *must be averaged* between the pixel amount length in the north-south direction and the pixel amount length in the east-west direction.

If the scale factor has not been obtained from a known length placed near to the desired feature on the same photo (i.e. having the same tilted deformation), it is necessary to correct the measure applying a *correction factor* (*CF*) that accounts for the inclination angle f of the observed feature with respect to the observer's direction, becoming

$$CF = \frac{1}{\cos f} \qquad \textbf{(2)}$$

If the lunar feature is placed at the position P (latitude b_P, longitude L_P) and the Moon's current center disc (C) is placed at the position (latitude b_C, longitude L_C) (that is, the *current libration in latitude* being b_C and the *current libration in longitude* being L_C), the angle f *at the center of the Moon* between P and C is:

$$\cos f = \cos b_P \cos b_C \cos(L_P - L_C) + \sin b_P \sin b_C \qquad \textbf{(3)}$$

Therefore, the *feature physical length* (*FPL*) becomes

$$FPL = m\,(SF)(CF) = \frac{m(SF)}{\cos f} \tag{4}$$

The "***drift timing method***" is based on the measurement of the feature angular size by timing its drifting passage through the crosshair eyepiece of the telescope and then applying the proper conversion to the measured angle in arc seconds.

As the Moon noticeable moves with respect to the stars, the lunar area viewed through the telescope eyepiece results *constantly being displaced* with the same angular speed as the Moon is currently moving. The lunar surface drift speed respect to the telescope sidereal tracking is roughly about 1 km/s in the Moon’s east-west direction.

The obtained *timing value* (t) in seconds multiply by the *Moon's angular speed at that moment* (v) in arcsec/sec gives the *feature angular size* expressed in arcsec, as seen by the observer.

According to Kepler's second law the orbital speed of the Moon is not constant, so its angular speed (which averages 0.549 arcsec/sec), also continually varies depending on its *current orbital position*.

The precise *current angular speed of the Moon* can be obtained from knowing the variations of both *right ascension* (ΔRA) and *declination* (ΔDEC) in an *interval of time* (ΔT) around that moment, being

$$v = \sqrt{\left(\frac{\Delta RA}{\Delta T}\right)^2 + \left(\frac{\Delta DEC}{\Delta T}\right)^2} \tag{5}$$

In order to convert the *feature angular size* (t times v) to a physical size in meters, it is necessary to apply the small-angle formula, multiplying by the *current Earth-Moon distance* (d_{EM}) in meters and dividing by the factor 206,265.

In the same way of the previous method, the obtained length must be corrected with regarding to the feature surface actual position (CF).

Finally, the *feature physical length* (FPL) becomes

$$FPL = t\,v\,\frac{d_{EM}}{206{,}265}(CF) = t\,v\,\frac{d_{EM}}{206{,}265}\,\frac{1}{\cos f} \tag{6}$$

b) Measuring lunar depths and heights

The depth or the height of any lunar feature below or above its surrounding surface can be obtained from measuring the shadow length cast by the Sun and then applying some calculation based on the obtained measure plus some lunar data specifically corresponding *to the same moment of the measurement*.

The theoretical method later on used in this project is a simplified version of the original "*method of projected shadows*", developed by Heinrich Olbers around 1780. Among the pieces of information important to the calculation are the Earth-Moon currently distance and the coordinates of three lunar surface points: the coordinates (b_P, L_P) of the feature peak (P), the coordinates (b_S, L_S) of the currently subsolar point (S) -the point at the lunar surface where the Sun was placed at its zenith for that particularly moment-, and the also currently coordinates (b_C, L_C) of the Moon's center disc (C) as seen by the observer.

Knowing all that information it will be possible to find out two necessary angles: the solar altitude (A) at the peak's position and the angle (y) between the Earth and the Sun as seen from the center of the Moon.

The simplified method consists in the following proceeding:

i) The *shadow apparent length* (SAL), in arcseconds, can be measured applying any of the two described former methods. In the usual case of the *drift timing method*, being t the measured time in seconds and v the current Moon's angular speed in arcsec/sec, it becomes

$$SAL = t\,v \qquad \textbf{(7)}$$

ii) The *current Moon's radii* (R_M), in arcseconds, is calculated from

$$R_M = \frac{(206{,}265)(1{,}738)}{d_{EM}} = \frac{(206{,}265)(1{,}738)}{(d_{EM}/R_E)(6{,}378)} = \frac{56{,}207}{d_{EM}/R_E} \qquad \textbf{(8)}$$

where (d_{EM}/ R_E) is the Earth-Moon distance expressed in Earth radii.

iii) The *shadow apparent length* (SAL) in terms of the *current Moon's radii* (R_M) becomes

$$s = \frac{(SAL)}{R_M} \qquad \textbf{(9)}$$

iv) The angle (y) between the Earth and the Sun as seen from the center of the Moon is the angle between the subsolar point (S) and the apparent center point (C) and therefore may be found from

$$\cos y = \cos b_S \cos b_C \cos(L_S - L_C) + \sin b_S \sin b_C \qquad \textbf{(10)}$$

v) The *unprojected shadow length as a fraction of the current Moon's radii* (USF) corresponds to the former *shadow apparent length* after being corrected for the foreshortening introduced because of the angle y . Hence, considering the perpendicular plane to the observer, the *unprojected shadow length fraction* becomes

$$USF = \frac{s}{\sin y} = \frac{(SAL)/R_M}{\sin y} \qquad \textbf{(11)}$$

vi) The Sun's altitude (A) at the feature at that precise time of the measurement is the complementary angle between P and S at the center of the Moon, so the altitude (A) is obtained from

$$\sin A = \cos b_P \cos b_S \cos(L_P - L_S) + \sin b_P \sin b_S \qquad \textbf{(12)}$$

vii) The feature height fraction (*FHF*), that is the actual feature height but expressed as a fraction of the *current Moon's radii*, finally results

$$FHF = (USF) \sin A - \frac{1}{2}(USF)^2 \cos^2 A - \frac{1}{8}\,(USF)^4 \cos^4 A \qquad (\mathbf{13})$$

viii) Therefore the *physical feature height* (*H*) in kilometers is

$$H = (FHF)\,(1{,}738) \qquad (\mathbf{14})$$

c) Evaluation of the involved theoretical accuracy in calculating *FPLs*

The overall theoretical error of any multiplication product, like the final formulae (**4**) and (**6**), corresponds to each error term times the other error terms.

The *correction factor* (*CF*) of both (**4**) and (**6**) actually introduces a very little theoretical error, as for its value it has been assumed two simplifications: firstly, the coordinates of the Moon's center should be *topographic* (corresponding to the observer's location on the Earth's surface) instead of the geocentric common used values of both librations, and secondly, the used angle f at the Moon's center between the Earth's center and the location of the lunar surface feature (*P*) is slightly different from the true angle *seen by the observer* between the Moon's center disc (*C*) and the location of the feature (*P*), difference that could be as much as the Moon's radii angular size -about 1/4 degree-.

Being the *correction factor* theoretical error so minimum, the overall accuracy of the *feature physical length* (*FPL*) obtained by the *photographic method* (**4**) basically depends on the accuracy of the *scale factor* (*SF*), assuming that the photo can achieve all the required resolution.

The final accuracy of the feature physical length (*FPL*) obtained by the *drift timing method* applying (**6**), besides the already commented inaccuracy of the factor (*CF*) and the imprecision of the instant *angular speed v*, also introduces another theoretical error due to the use of the Earth-Moon distance (d_{EM}) instead of the proper *observer-feature distance*. Therefore the easily found d_{EM} value is always a little greater that the actual value that should be used, difference that could be as much as the Earth's radii plus the Moon's radii -about as 2% of d_{EM}-.

Therefore, the overall theoretical error of both here used methods are well above 3%. Anyway, the likely instrumental and human errors in the measurement are a lot much greater than the error generated by the former simplifications of both the angle f and the distance d_{EM}.

However, if precision is a must, both the *exact angle at the feature* between its zenith and the observer (approximately f) and the *true* currently distance between the observer and the lunar surface feature (approximately d_{EM}) can be calculated from knowing the following data *with respect to the observer*: his Earth's coordinates and local hour angle (which allows calculate the exact current lunar altitude) and his true *topocentric* coordinates of the apparent center of the Moon's disc.

d) Evaluation of the involved theoretical accuracy in calculating *FHFs*

In obtaining the *feature physical height* (*FPH*) final formula (**13**), besides the same considerations done in the previous topic with respect to the two simplifications f and d_{EM}, it has been used the angle y instead of the true angle between the observer and the Sun *at the point P* (known as the "*local phase angle*").

The overall theoretical error of this simplified version of the "*method of projected shadows*" is less than about 3.5% for the worst case.

2) Applied measurement procedures

a) Equipment and location

All the field measurements has been obtained using the same telescope and from the same place, on several days during May and June of 2002. The observational place was located at Salto, Uruguay, with coordinates 31° 23.43' south, 57° 58.97' west, elevation cote 28 m, official time zone - 4 hours (but currently using daylight saving time all year long).

The used telescope was an 8" Schmidt-Cassegrain (Meade LX90), 2000mm, f/10, electronic sidereal tracking, mounted on an altazimuth tripod. The used eyepieces were Meade Super Plossl series 4000 (40mm, 26mm, 15mm and 9.7mm), a Meade Modified Achromatic 12mm illuminated reticle, and a Meade 2x Apochromatic Barlow lens series 4000. Alternatively, it was used a digital camera (Sony MAVICA MVC-FD71), f = 4.2 - 42 mm, F 1.8 - 2.9, and a digital stopwatch (Seiko S031).

Photo N° 1
Telescope Meade LX-90 at field with dew shield

Photo N° 2
Set of eyepieces and the digital stopwatch

b) Description of the measurement applied proceedings

For each measure it was recorded the *local time*, the *Moon's altitude* obtained from the telescope altitude circle (with a resolution of 0.5 degree), the *name of the feature* and type of measurement (*size or height*), the *drift timing measure* (when applied), and alternatively it was made a sketch or it was taken a digital photo of the observed place.

All the photos were taken with the Mavica Sony digital camera pointing directly at the image formed at the telescope eyepiece, placing the camera objective very close to the eyepiece. The photographs were taken in the autofocus position, fine quality, JPEG format, 640 x 480 pixels.

The photographs was taken trying to reach the highest possible quality compatible with the greater practical amplification, with features clearly defined in order to enable accurate measurements.

The telescope was always used in its *automatic sidereal tracking rate.* As the manufacturer suggests to check the telescope pointing accuracy once every six months, that task was performed by mid May, previous to the beginning of the observational field work.

One axis of the crosshair eyepiece was always placed in right angle to the lunar terminator (*basically the direction of the lunar surface drifting*), so the timing measurements were done with respect at the other axis (that seemed to be displaced parallel to the terminator).

In order to achieve the maximum possible accuracy, all the drift timing measures were obtained working with the 12mm crosshair eyepiece plus the Barlow lens, that is reaching a magnification of 333X. Besides having a better defined border of each feature size or shadow, the greater the magnification, the lesser the appreciation error due to the fixed width of the crosshair axis.

Due to the important dispersion values found when the drift timing method was applied (the afterwards computed *dispersion values* appear next to the right of each timing measure) each feature measure was repeated several times for improving the accuracy of the results, actually performing *series of measurements*. As the lunar shadows rapidly change their lengths, especially with the terminator nearby, the shadow length series measurements were done trying to get all the measures *within the shortest possible interval* (which anyway always became greater than 20 minutes).

The chosen feature peaks shadows to be measured were finally selected because of two reasons: the peaks were currently close enough to the terminator as to present *a long and well-lit* shadow (meaning a solar altitude between 2° and 10°), and also the surrounding terrain where the shadow fell seemed to be *flat enough* as to not introduce any additional measurement errors.

In two different cases of height measurements it was possible to measure the shadow lengths projected to opposite directions, once to the west and a fortnight later projected to the east.

When the obtained drift timing measures were greatly different from the rest of the series values due to anomalous reasons (which actually happened for a few cases), those measure values were eliminated.

The experimental dispersion in the obtained timing values shows that:
i) for all the crater size measurements, the percentage difference was better than 10%;
ii) for all the shadows length measurements, the percentage difference was better than 18%, notoriously increasing the dispersion with lesser timing values.

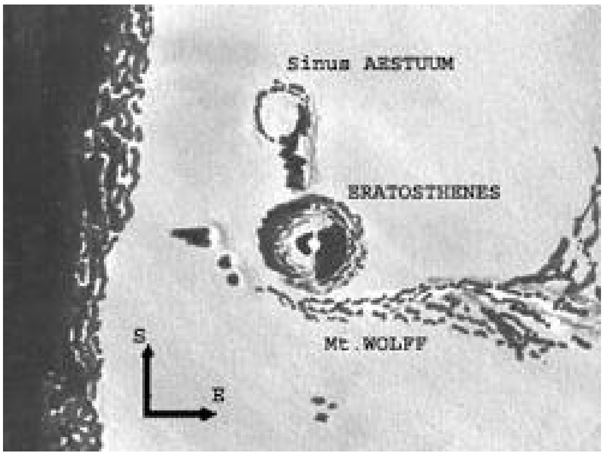
3) Observing sessions

Observation # 1

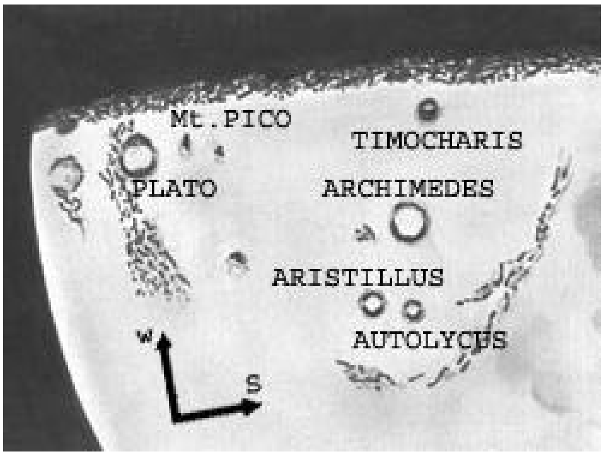
Date: 20 May 2002, Initial local time: 18:15 hs, Final local time: 21:18 hs
Weather conditions: 19°C, 1009 hPa, 60% rel
Sky conditions: quite clear
Moon's phase: Waxing gibbous (9 days)

Observed features:

N°	Loc. Time	Moon's altitude	Measured feature	Drift timing	Dispersion
1	19:09	43.5°	Crater Eratosthenes size	53.25 sec	0 %
2	19:13	44.0°	Crater Eratosthenes size	50.14 sec	- 6 %
3	19:25	45.5°	Crater Eratosthenes size	56.88 sec	+ 6 %
4	20:23	47.0°	Mons Pico heigth	23.07 sec	- 7 %
5	20:43	46.5°	Mons Pico heigth	28.15 sec	+ 14 %
6	20:53	45.5°	Mons Pico heigth	22.81 sec	- 8 %



Sketch N° 1
19:50 hs, Eratosthenes, colongitude = 18°



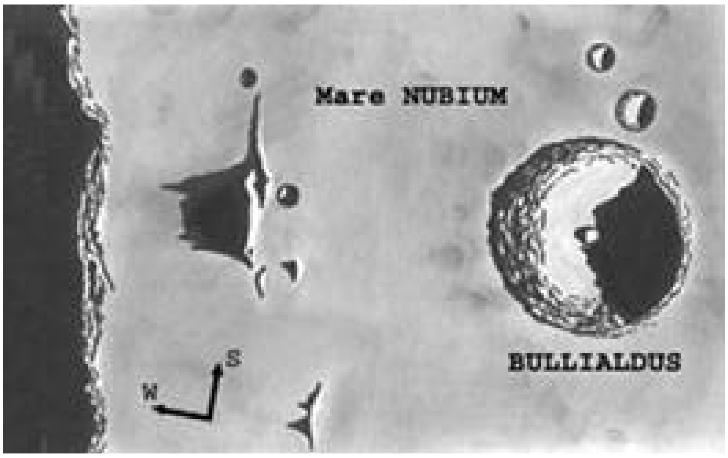
Sketch N° 2
20:12 hs, Plato & nearby Mons Pico, colong=18°

Observation # 2

Date: 21 May 2002, Initial local time: 18:29 hs, Final local time: 20:31 hs
Weather conditions: 20°C, 1009 hPa, 63% rel
Sky conditions: quite clear
Moon's phase: Waxing gibbous (10 days)

Observed features:

N°	Loc.Time	Moon's altitude	Measured feature	Drift timing	Dispersion
1	18:59	43.0°	Crater Bullialdus size	58.56 sec	0 %
2	19:17	46.0°	Crater Bullialdus size	55.32 sec	-6 %
3	19:21	46.0°	Mons west to Bullialdus heigth	33.23 sec	+ 3 %
4	19:35	46.5°	Crater Bullialdus size	60.09 sec	+ 3 %
5	19:39	47.0°	Mons west to Bullialdus heigth	29.68 sec	- 8 %
6	20:13	50.5°	Crater Bullialdus size	57.08 sec	- 3 %
7	20:16	51.0°	Mons west to Bullialdus heigth	33.75 sec	+ 5 %
8	20:19	51.0°	Crater Bullialdus size	61.69 sec	+ 5 %



Sketch N° 3
20:00 hs, Bullialdus and Mons to its west, colong=30°

Observation # 3

Date: 22 May 2002, Initial local time: 18:45 hs, Final local time: 21:30 hs
Weather conditions: 19°C, 1006 hPa, 68% rel
Sky conditions: quite clear
Moon's phase: Waxing gibbous (11 days)

Observed features:

N°	Loc. Time	Moon's altitude	Measured feature	Drift timing	Dispersion
1	19:22	44.0°	Crater Gassendi size	83.19 sec	+ 3 %
2	19:30	45.0°	Crater Gassendi size	86.09 sec	+ 7 %
3	19:35	45.5°	Crater Gassendi size	76.28 sec	- 5 %
4	19:39	47.0°	Crater Gassendi size	72.63 sec	- 10 %
5	19:45	47.5°	Crater Gassendi size	84.90 sec	+ 5 %
6	20:33	54.0°	Promontorium Kelvin heigth	7.11 sec	- 18 %
7	20:57	57.0°	Promontorium Kelvin heigth	9.61 sec	+ 11 %
8	21:08	57.5°	Promontorium Kelvin heigth	7.97 sec	- 8 %
9	21:17	58.0°	Promontorium Kelvin heigth	10.14 sec	+17 %
10	21:23	58.5°	Promontorium Kelvin heigth	8.33 sec	- 3 %

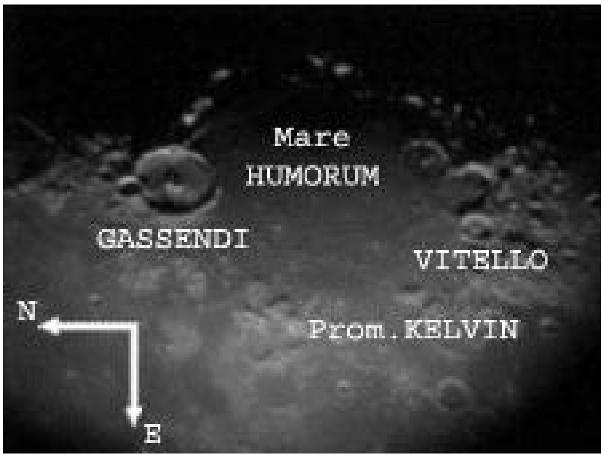


Photo N° 3
19:08 hs, 2x15 mm, 0 EV, colongitude=42°

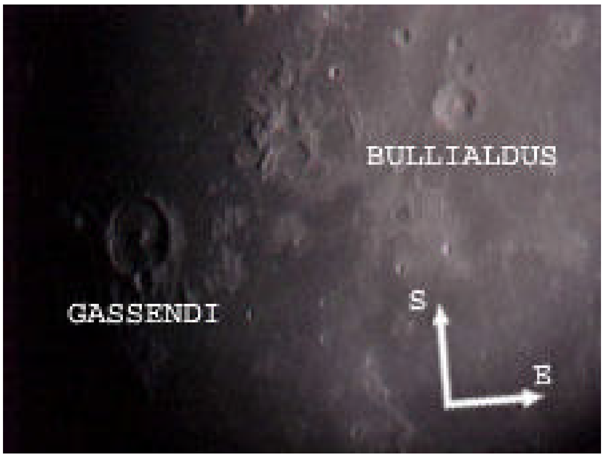


Photo N° 4
19:12 hs, 2x15 mm, 0 EV, colongitude=42°

Observation # 4

Date: 03 June 2002, Initial local time: 03:11 hs, Final local time: 05:45 hs
Weather conditions: 20°C, 1010 hPa, 80% rel
Sky conditions: acceptably clear until became almost totally overcast
Moon's phase: Last quarter (22 days)

Observed features:

N°	Loc. Time	Moon's altitude	Measured feature	Drift timing	Dispersion
1	04:34	44.5°	Crater Plato size	82.40 sec	- 1 %
2	04:38	45.0°	Crater Plato size	79.20 sec	- 5 %
3	04:42	47.0°	Crater Plato size	80.81 sec	- 3 %
4	04:48	47.5°	Crater Plato size	85.17 sec	+ 3 %
5	04:51	48.0°	Crater Plato size	87.53 sec	+ 5 %
6	04:58	49.5°	Mons Pico heigth	24.72 sec	- 1 %
7	05:05	50.5°	Mons Pico heigth	28.47 sec	+ 14 %
8	05:12	51.5°	Mons Pico heigth	21.96 sec	- 12 %
9	05:19	54.0°	Mons Pico heigth	24.82 sec	- 1 %
10	05:25	55.0°	Mons Pico heigth	25.30 sec	+ 1 %

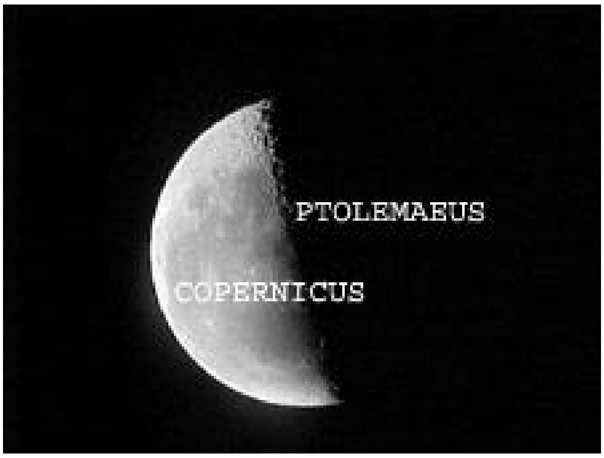


Photo N° 5
03:33 hs, 40 mm, -1.5 EV, B&W, colong=181°

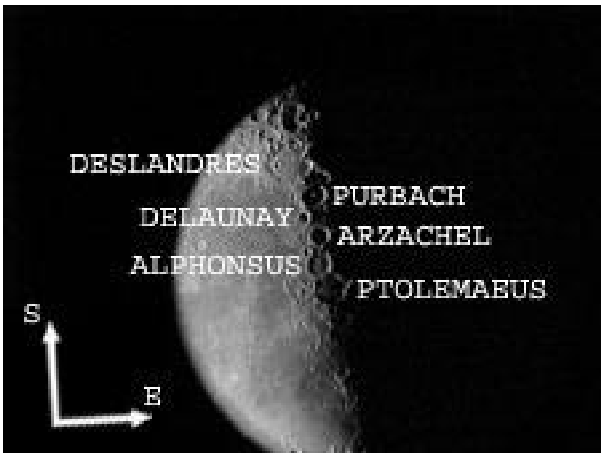


Photo N° 6
03:38 hs, 2x26 mm, -1.5 EV, B&W, colong=181°

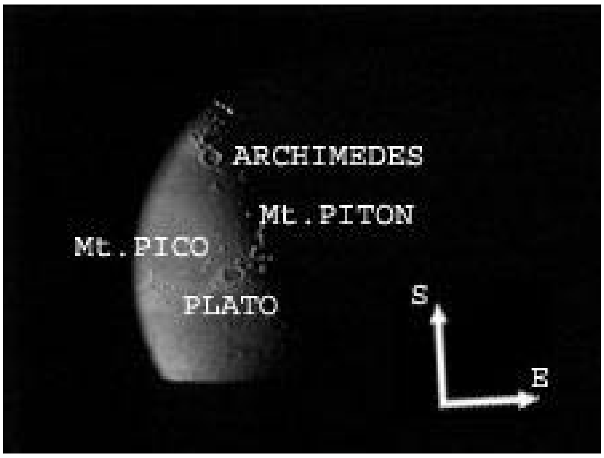


Photo N° 7
04:10 hs, 26 mm, -1.5 EV, B&W, colong=181°

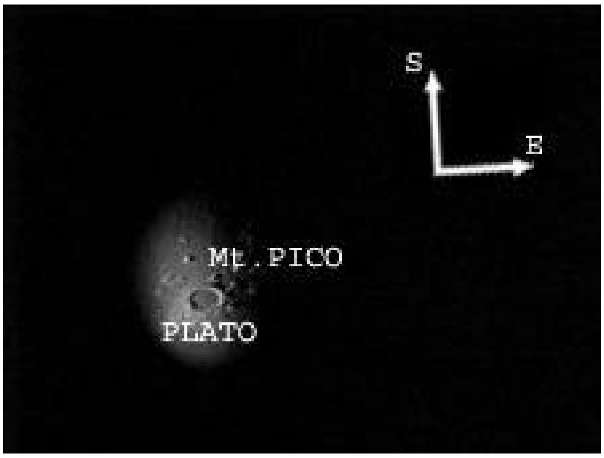


Photo N° 8
04:29 hs, 12 mm, -1.5 EV, B&W, colong=181°

Observation # 5

Date: 05 June 2002, Initial local time: 04:15 hs, Final local time: 06:58 hs
Weather conditions: 22°C, 1003 hPa, 85% rel
Sky conditions: partly cloudy
Moon's phase: Waning crescent (24 days)

Observed features:

N°	Loc. Time	Moon's altitude	Measured feature	Drift timing	Dispersion
1	05:30	34.0°	Promontorium Kelvin heigth	15.16 sec	- 7 %
2	05:38	35.5°	Promontorium Kelvin heigth	17.20 sec	+ 5 %
3	05:45	37.0°	Promontorium Kelvin heigth	18.76 sec	+ 15 %
4	05:49	38.0°	Promontorium Kelvin heigth	14.58 sec	- 11 %
5	05:53	38.5°	Promontorium Kelvin heigth	16.02 sec	- 2 %
6	06:25	44.5°	Crater Gassendi size	86.26 sec	+ 5 %
7	06:29	45.0°	Crater Gassendi size	84.59 sec	+ 3 %
8	06:34	45.5°	Crater Gassendi size	78.14 sec	- 5 %
9	06:37	46.0°	Crater Gassendi size	80.46 sec	- 2 %
10	06:40	46.5°	Crater Gassendi size	81.17 sec	- 1 %

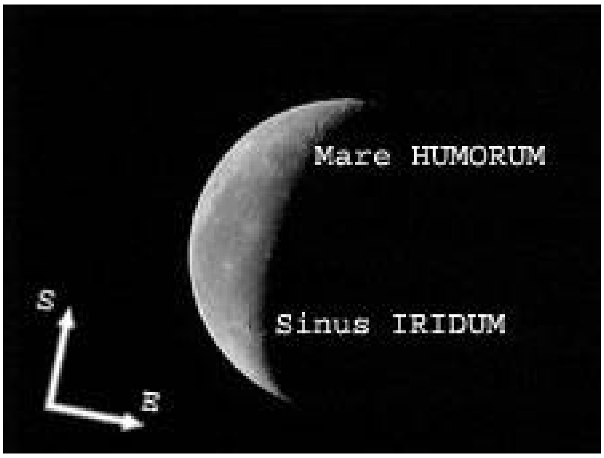


Photo N° 9
04:39 hs, 40 mm, -1.5 EV, B&W, colong=206°

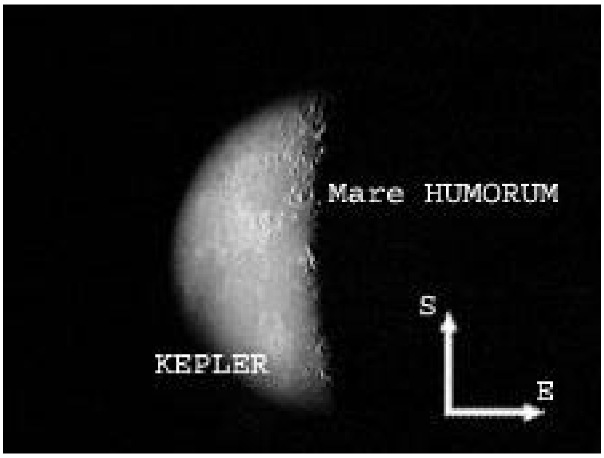


Photo N° 10
06:01 hs, 26 mm, -1.5 EV, B&W, colong=206°

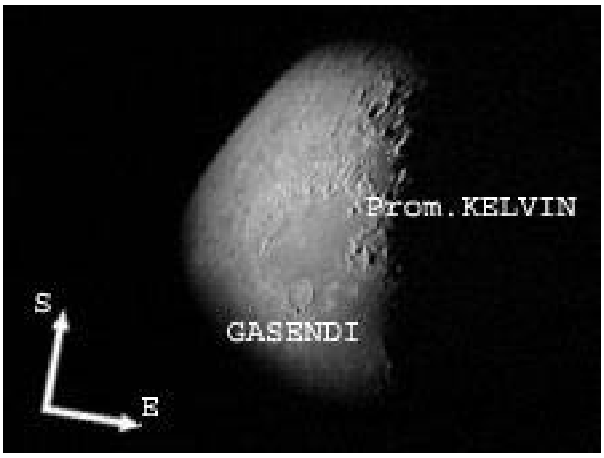


Photo N° 11
06:06 hs, 15 mm, -1.5 EV, B&W, colong=206°

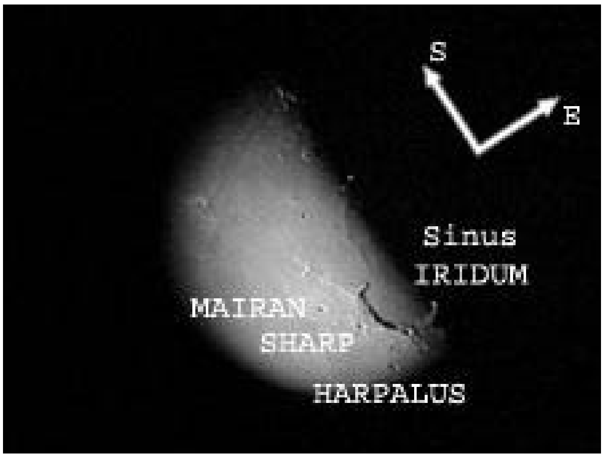


Photo N° 12
06:46 hs, 15 mm, -1.5 EV, B&W, colong=207°

Observation # 6

Date: 06 June 2002, Initial local time: 04:45 hs, Final local time: 06:58 hs
Weather conditions: 15°C, 1021 hPa, 65% rel
Sky conditions: slightly hazy
Moon's phase: Waning crescent (25 days)

Observed features:

N°	Loc. Time	Moon's altitude	Measured feature	Drift timing	Dispersion
1	05:28	21.5°	Crater Gassendi size	86.36 sec	+ 4 %
2	05:34	25.0°	Crater Gassendi size	79.80 sec	- 3 %
3	05:43	25.5°	Crater Gassendi size	78.51 sec	- 5 %
4	06:12	31.0°	Crater Gassendi size	83.66 sec	+ 1 %
5	06:19	32.5°	Crater Gassendi size	85.10 sec	+ 3 %

Photo N° 13
05:10 hs, 40 mm, -1.5 EV, B&W, colong=218°

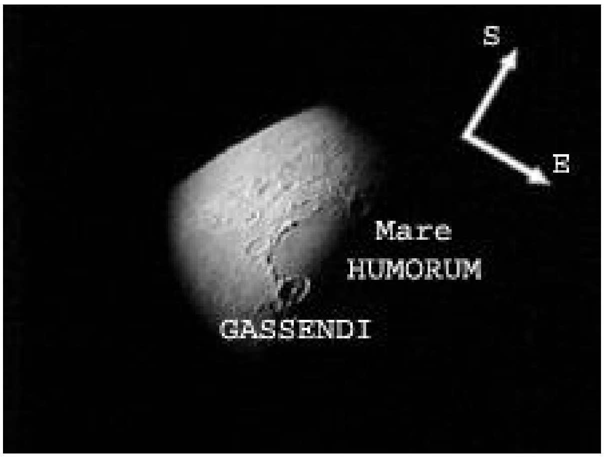


Photo N° 14
05:13 hs, 15 mm, -1.5 EV, B&W, colong=218°

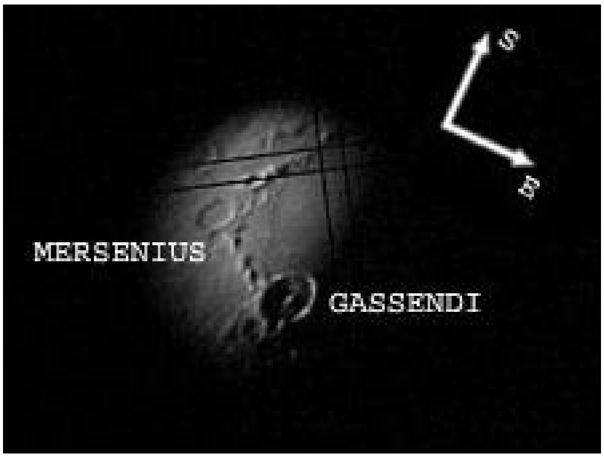


Photo N° 15
06:01 hs, 12 mm, -1.5 EV, B&W, colong=219°

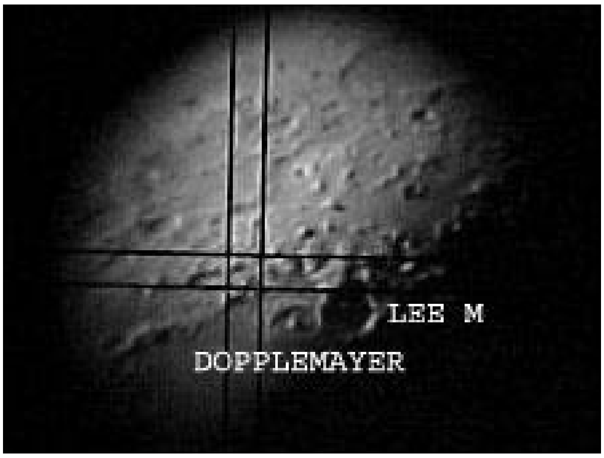


Photo N° 16
05:58, 12 mm, -1.5 EV, colong=219°

4) Acquisition of related parameters

The physical ephemerides of the Moon for each date and time of the six observations have been obtained from http://www.xylem.demon.co.uk/kepler/jsmoon.html.

The corresponding extracted data are the following:

Physical ephemeris of the Moon for the observation # 1	**20 May 2002 22:01 UT**	**20 May 2002 23:00 UT**	**21 May 2002 00:00 UT**
Julian day number	2,452,415.4174	2,452,415.4583	2,452,415.5000
Libration in latitude (°)	-6.8	-6.8	-6.8
Libration in longitude (°)	-3.1	-3.0	-3.0
Colongitude of the Sun (°)	17.71	18.21	18.72
Subsolar point latitude (°)	-0.5	-0.5	-0.5
Subsolar point longitude (°)	72.3	71.8	71.3
Distance Earth-Moon (Earth radii)	57.80	57.79	57.77
Right ascension (hours)	11.169	11.205	11.242
Declination (°)	11.06	10.83	10.60

Physical ephemeris of the Moon for the observation # 2	**21 May 2002 22:01 UT**	**21 May 2002 22:30 UT**	**21 May 2002 23:00 UT**
Julian day number	2,452,416.4174	2,452,416.4375	2,452,416.4583
Libration in latitude (°)	-6.7	-6.6	-6.6
Libration in longitude (°)	-1.8	-1.8	-1.7
Colongitude of the Sun (°)	29.91	30.15	30.41
Subsolar point latitude (°)	-0.5	-0.5	-0.5
Subsolar point longitude (°)	60.1	59.8	59.6
Distance Earth-Moon (Earth radii)	57.48	57.47	57.46
Right ascension (hours)	12.053	12.070	12.089
Declination (°)	5.26	5.14	5.01

Physical ephemeris of the Moon for the observation # 3	**22 May 2002 22:30 UT**	**22 May 2002 23:30 UT**	**23 May 2002 00:30 UT**
Julian day number	2,452,417.4375	2,452,417.4792	2,452,417.5208
Libration in latitude (°)	-6.1	-6.0	-6.0
Libration in longitude (°)	-0.4	-0.3	-0.2
Colongitude of the Sun (°)	42.35	42.86	43.37
Subsolar point latitude (°)	-0.5	-0.5	-0.5
Subsolar point longitude (°)	47.7	47.1	46.6
Distance Earth-Moon (Earth radii)	57.26	57.26	57.25
Right ascension (hours)	12.948	12.985	13.021
Declination (°)	-1.01	-1.27	-1.53

Physical ephemeris of the Moon for the observation # 4	**03 Jun 2002 06:30 UT**	**03 Jun 2002 07:30 UT**	**03 Jun 2002 08:30 UT**
Julian day number	2,452,428.7708	2,452,428.8125	2,452,428.8542
Libration in latitude (°)	6.8	6.8	6.8
Libration in longitude (°)	2.1	2.0	2.0
Colongitude of the Sun (°)	180.55	181.06	181.57
Subsolar point latitude (°)	-0.1	-0.1	-0.1
Subsolar point longitude (°)	269.4	268.9	268.4
Distance Earth-Moon (Earth radii)	63.30	63.31	63.32
Right ascension (hours)	23.238	23.269	23.300
Declination (°)	-10.66	-10.46	-10.27

Physical ephemeris of the Moon for the observation # 5	**05 Jun 2002 07:30 UT**	**05 Jun 2002 08:30 UT**	**05 Jun 2002 09:30 UT**
Julian day number	2,452,430.8125	2,452,430.8542	2,452,430.8958
Libration in latitude (°)	6.4	6.3	6.3
Libration in longitude (°)	-0.4	-0.5	-0.6
Colongitude of the Sun (°)	205.51	206.02	206.53
Subsolar point latitude (°)	-0.1	-0.1	-0.1
Subsolar point longitude (°)	244.5	244.0	243.5
Distance Earth-Moon (Earth radii)	63.38	63.38	63.37
Right ascension (hours)	0.719	0.749	0.779
Declination (°)	-0.72	-0.51	-0.30

Physical ephemeris of the Moon for the observation # 6	**06 Jun 2002 08:01 UT**	**06 Jun 2002 09:01 UT**	**06 Jun 2002 10:00 UT**
Julian day number	2,452,431.8340	2,452,431.8757	2,452,431.9167
Libration in latitude (°)	5.7	5.7	5.6
Libration in longitude (°)	-1.7	-1.7	-1.8
Colongitude of the Sun (°)	218.01	218.52	219.02
Subsolar point latitude (°)	-0.1	-0.1	-0.1
Subsolar point longitude (°)	232.0	231.5	231.0
Distance Earth-Moon (Earth radii)	63.10	63.08	63.07
Right ascension (hours)	1.456	1.487	1.517
Declination (°)	4.41	4.62	4.82

Coordinates and actual sizes of lunar features have also been obtained from the Internet.

5) Determination of the required sizes and heights

The lunar features sizes and heights have been calculated by means of the application of the observational field measures and its corresponding data (coordinates and ephemerides) to the proper formulae discussed at the beginning.

At the beginning of each calculation it is indicated which was the used proceeding for obtaining the corresponding field measure: ***PM*** for the *photographic method*, ***DTM*** for the *drift timing method*.

The detailed proceeding is only described just for the first cases of each method.

For each series measurement, the Moon's current numbers were taken from just one single time, so that the slight change in the ephemerides values through all each series measurement interval has been ignored.

The current values of the geocentric *libration in latitude* and *libration in longitude* were taken as the corresponding coordinates (b_C, L_C) of the current apparent center of the Moon's disc C.

Observation # 1

DTM: The size of the crater Eratosthenes (b_P = 14.5° N, L_P = 11.3° W)

The average drift timing (t) of the three measurements results 53.4 seconds.

In order to find out the angular speed of the Moon at the moment of the measurements (taking the interval between 22:01 UT and 23:00 UT), from the Moon's ephemeris it is obtained

ΔRA (h) = (11.205 - 11.169) h = 0.036 h
ΔRA (°) = 15 °/h x ΔRA (h) = 15°/h x 0.036 h = 0.54 °
ΔDEC (°) = (10.83 - 11.06) ° = - 0.23 °
ΔT = 59 min = 0.983 h

Hence, according to (**5**)

$$v = \sqrt{\left(\frac{\Delta RA}{\Delta T}\right)^2 + \left(\frac{\Delta DEC}{\Delta T}\right)^2} = \sqrt{\left(\frac{0.54}{0.983}\right)^2 + \left(\frac{0.23}{0.983}\right)^2} = 0.597\ °/\text{h}$$

v = 0.597 arcsec/sec

The correction factor (CF) is calculated from (**3**)

$$\cos f = \cos b_P \cos b_C \cos(L_P - L_C) + \sin b_P \sin b_C$$

$$\cos f = \cos(14.5)\cos(-6.8)\cos[(-11.3) - (-3.1)] + \sin(14.5)\sin(-6.8) = 0.922$$

From (**6**) results

$$FPL_{Eratosthenes} = (53.4)(0.597)\frac{(57.80)(6{,}378)}{206{,}265}\frac{1}{0.922} = 61.8 \text{ km}$$

DTM: The heigth of the Mons Pico (1) (b_P = 45.7° N, L_P = 8.9° W)

On average, t = 24.7 seconds

For the Moon's ephemeris of the 23:00 - 00:00 UT interval, applying (**5**) it is obtained that v = 0.601 arcsec/sec, and from (**7**) results SAL = 14.84 arcsec

Taking d_{EM} = 57.79 R_E, from (**8**) R_M = 972.6 arcsec and from (**9**) s = 0.01526

The shadow correction angle y is calculate from (**10**)

$$\cos y = \cos b_S \cos b_C \cos(L_S - L_C) + \sin b_S \sin b_C$$

$$\cos y = \cos(-0.5)\cos(-6.8)\cos[71.8 - (-3.0)] + \sin(-0.5)\sin(-6.8) = 0.261$$

$$y = 74.8\ °$$

Hence from (**11**)

$$USF = \frac{s}{\sin y} = \frac{0.01526}{\sin 74.8} = 0.01581$$

The Sun's altitude calculated from (**12**) becomes

$$\sin A = \cos b_P \cos b_S \cos(L_P - L_S) + \sin b_P \sin b_S$$

$$\sin A = \cos(45.7)\cos(-0.5)\cos(-8.9 - 71.8) + \sin(45.7)\sin(-0.5) = 0.107$$

$$A = 6.1\ °$$

Finally from (**13**)

$$FHF_{Pico(1)} = (USF)\sin A - \frac{1}{2}(USF)^2\cos^2 A - \frac{1}{8}(USF)^4\cos^4 A$$

$$FHF_{Pico(1)} = (0.01581)\sin(6.1) - \frac{1}{2}(0.01581)^2\cos^2(6.1) - \frac{1}{8}(0.01581)^4\cos^4(6.1)$$

$$FHF_{Pico(1)} = 0.001556$$

Therefore

$$H_{Pico(1)} = (0.001556)(1{,}738) = 2.71 \text{ km}$$

Observation # 2

DTM: The size of the crater Bullialdus (b_P = 20.7° S, L_P = 22.2° W)
On average, t = 58.5 seconds
It is obtained from (**5**) v = 0.605 arcsec/sec, and from (**3**) $\cos f =$ 0.912
Resulting

$$FPL_{Bullialdus} = (58.5)(0.605)\frac{(57.47)(6{,}378)}{206{,}265}\frac{1}{0.912} = 69.0 \text{ km}$$

DTM: The heigth of the mons west to Bullialdus (b_P = 20.7° S, L_P = 27.3° W ?)
On average, t = 32.2 seconds
It is obtained from (**5**) v = 0.605 arcsec/sec, so from (**7**) results SAL = 19.48 arcsec
Taking d_{EM} = 57.47 R_E, from (**8**) R_M = 978.0 arcsec and from (**9**) s = 0.01992
The shadow correction angle y is calculate from (**10**)

$$\cos y = \cos b_S \cos b_C \cos(L_S - L_C) + \sin b_S \sin b_C$$
$$\cos y = \cos(-0.5) \cos(-6.6) \cos[59.8 - (-1.8)] + \sin(-0.5) \sin(-6.6) = 0.4735$$
$$y = 61.7\,°$$

Hence from (**11**) $USF = \dfrac{s}{\sin y} = 0.02262$

The Sun's altitude calculated from (**12**) becomes

$$\sin A = \cos b_P \cos b_S \cos(L_P - L_S) + \sin b_P \sin b_S$$
$$\sin A = \cos(-20.7) \cos(-0.5) \cos(-27.3 - 59.8) + \sin(-20.7) \sin(-0.5) = 0.0504$$
$$A = 2.9\,°$$

Finally from (**13**)

$$FHF_{Mons(?)} = (USF) \sin A - \frac{1}{2}(USF)^2 \cos^2 A - \frac{1}{8}(USF)^4 \cos^4 A$$
$$FHF_{Mons(?)} = (0.02262) \sin(2.9) - \frac{1}{2}(0.02262)^2 \cos^2(2.9) - \frac{1}{8}(0.02262)^4 \cos^4(2.9)$$
$$FHF_{Mons(?)} = 0.000889$$

Therefore

$$H_{Mons(?)} = (0.000889)(1{,}738) = 1.55 \text{ km}$$

Observation # 3

DTM: The size of the crater Gassendi (1) (b_P = 17.6° S, L_P = 40.1° W)
On average, t = 80.6 seconds; d_{EM} = 57.26 R_E
It is obtained from (**5**) v = 0.613 arcsec/sec, (**3**) $\cos f =$ 0.761, resulting

$$FPL_{Gassendi(1)} = (80.6)(0.613)\frac{(57.26)(6{,}378)}{206{,}265}\frac{1}{0.761} = 115.0 \text{ km}$$

DTM: The heigth of the Promontorium Kelvin (1) (b_P = 27.0° S, L_P = 33.0° W)
On average, t = 8.63 seconds
It is obtained from (**5**) v = 0.599 arcsec/sec, so from (**7**) results SAL = 5.17 arcsec
Taking d_{EM} = 57.26 R_E, from (**8**) R_M = 981.6 arcsec and from (**9**) s = 0.00527
The shadow correction angle y is calculate from (**10**) y = 47.6 °

Hence from (**11**) USF = 0.00714

The Sun's altitude calculated from (**12**) becomes $A = 9.0\ °$
Finally from (**13**)

$$FHF_{Kelvin(1)} = (0.00714)\sin(9.0) - \frac{1}{2}(0.00714)^2\cos^2(9.0) - \frac{1}{8}(0.00714)^4\cos^4(9.0)$$

$$FHF_{Kelvin(1)} = 0.001092$$

Therefore

$$H_{Kelvin(1)} = (0.001092)(1{,}738) = 1.90 \text{ km}$$

Observation # 4

PM: The size of the crater Ptolemaeus (b_P = 9.3° S, L_P = 1.9° W)
Working with the Microsoft Photo Editor software on the digital Photo N° 5, it is obtained the coordinates of the diameter of the Moon's image P_1 (282, 100) and P_2 (363, 424).
Applying (**1**)

$$\text{Moon}_{\text{Diameter}} \text{ (pixels)} = \sqrt{(363-282)^2 + (424-100)^2} = 334 \text{ pixels}$$

As the physical Moon's diameter is 3476 km, the scale factor (*SF*) becomes 10.41 km/pix
In the same way, the diameter of Ptolemaeus results m = 13.34 pix
From (**3**), $\cos f = 0.958$
Hence

$$FPL_{Ptolemaeus} = m\ (SF)\ (CF) = (13.34)(10.41)\frac{1}{0.958} = 145.0 \text{ km}$$

PM: The size of the crater Alphonsus (b_P = 13.7° S, L_P = 3.2° W)
From the Photo N° 6, the diameter of Alphonsus results m = 27.5 pix, and for the nearby crater Ptolemaeus m = 36.1 pix
Knowing from the former measure that $FPL_{Ptolemaeus}$ = 145.0 km, (*SF*)(*CF)* = 4.02 km/pix

$$FPL_{Alphonsus} = (27.5)(4.02) = 110.6 \text{ km}$$

DTM: The size of the crater Plato (b_P = 51.6° N, L_P = 9.4° W)
On average, t = 83.0 seconds
It is obtained from (**5**) v = 0.506 arcsec/sec, and from (**3**) $\cos f =$ 0.697
Resulting

$$FPL_{Plato} = (83.0)(0.506)\frac{(63.31)(6{,}378)}{206{,}265}\frac{1}{0.697} = 118.0 \text{ km}$$

PM: The heigth of the Mons Pico (2) (b_P = 45.7° N, L_P = 8.9° W)
From Photo N° 8, the shadow of Pico results m = 9.06 pix and the averaged diameter of the nearby Plato results m = 37.0 pix
The true FPL_{Plato} is 109 km, so (*SF*)(*CF*) = 2.95 km/pix
Therefore $SAL_{Pico(2)}$ = (9.06)(2.95) = 26.7 km
The shadow apparent length (*SAL*) in terms of the current Moon's radii (R_M), where both values must be expressed in the same units, becomes

$$s = \frac{(SAL)}{R_M} = \frac{26.7}{1{,}738} = 0.01536$$

The shadow correction angle y is calculate from (**10**) y = 93.1 °

Hence from (**11**) $USF = 0.01538$
The Sun's altitude calculated from (**12**) becomes $A = 5.4$ °
Finally from (**13**)

$$FHF_{Pico(2)} = (0.01538)\sin(5.4) - \frac{1}{2}(0.01538)^2\cos^2(5.4) - \frac{1}{8}(0.01538)^4\cos^4(5.4)$$

$$FHF_{Pico(2)} = 0.001330$$

Therefore

$$H_{Pico(2)} = (0.001330)(1{,}738) = 2.31 \text{ km}$$

<u>DTM: The heigth of the Mons Pico (3)</u> ($b_P = 45.7°$ N, $L_P = 8.9°$ W)
On average, $t = 25.1$ seconds
It is obtained from (**5**) $v = 0.506$ arcsec/sec, so from (**7**) results $SAL = 12.70$ arcsec
Taking $d_{EM} = 63.31\ R_E$, from (**8**) $R_M = 887.8$ arcsec and from (**9**) $s = 0.01431$
The shadow correction angle y is calculate from (**10**) $y = 93.1$ °
Hence from (**11**) $USF = 0.01433$
The Sun's altitude calculated from (**12**) becomes $A = 5.4$ °
Finally from (**13**)

$$FHF_{Pico(3)} = (0.01433)\sin(5.4) - \frac{1}{2}(0.01433)^2\cos^2(5.4) - \frac{1}{8}(0.01433)^4\cos^4(5.4)$$

$$FHF_{Pico(3)} = 0.001249$$

Therefore

$$H_{Pico(3)} = (0.001249)(1{,}738) = 2.17 \text{ km}$$

Observation # 5

<u>DTM: The heigth of the Promontorium Kelvin (2)</u> ($b_P = 27.0°$ S, $L_P = 33.0°$ W)
On average, $t = 16.3$ seconds
It is obtained from (**5**) $v = 0.497$ arcsec/sec, so from (**7**) results $SAL = 8.101$ arcsec
Taking $d_{EM} = 63.38\ R_E$, from (**8**) $R_M = 886.8$ arcsec and from (**9**) $s = 0.009135$
The shadow correction angle y is calculate from (**10**) $y = 115.3$ °
Hence from (**11**) $USF = 0.01010$
The Sun's altitude calculated from (**12**) becomes $A = 6.3$ °
Finally from (**13**)

$$FHF_{Kelvin(2)} = (0.01010)\sin(6.3) - \frac{1}{2}(0.01010)^2\cos^2(6.3) - \frac{1}{8}(0.01010)^4\cos^4(6.3)$$

$$FHF_{Kelvin(2)} = 0.001058$$

Therefore

$$H_{Kelvin(2)} = (0.001058)(1{,}738) = 1.84 \text{ km}$$

<u>DTM: The size of the crater Gassendi (2)</u> ($b_P = 17.6°$ S, $L_P = 40.1°$ W)
On average, $t = 82.1$ seconds; $d_{EM} = 63.38\ R_E$
It is obtained from (**5**) $v = 0.497$ arcsec/sec, (**3**) $\cos f = 0.733$
Resulting

$$FPL_{Gassendi(2)} = (82.1)(0.497)\frac{(63.38)(6{,}378)}{206{,}265}\frac{1}{0.733} = 109.1 \text{ km}$$

Observation # 6

DTM: The size of the crater Gassendi (3) (b_P = 17.6° S, L_P = 40.1° W)
On average, t = 82.7 seconds; d_{EM} = 63.10 R_E
It is obtained from (**5**) v = 0.510 arcsec/sec, (**3**) $\cos f = 0.752$
Resulting

$$FPL_{Gassendi} = (82.7)(0.510)\frac{(63.10)(6{,}378)}{206{,}265}\frac{1}{0.752} = 109.4 \text{ km}$$

PM: The size of the crater Gassendi (4) (b_P = 17.6° S, L_P = 40.1° W)
From the Photo N° 13, $\text{Moon}_{\text{Diameter}}$ = 328 pixels, so (SF) = 10.6 km/pix
The measure of Gassendi's size according to the orientation north-south is m_{NS} = 8.60 pix, and in the orientation east-west is m_{EW} = 7.21 pix, hence the average is m = 7.9 pix
Therefore

$$FPL_{Gassendi} = (7.9)(10.6)\frac{1}{0.752} = 111.4 \text{ km}$$

6) Comparison of the calculated values with published data

The following table summarizes each lunar feature measurement that has been done in this project, including the used method, its final calculated value, the corresponding actual physical value extracted from published data and the percentage difference between both values.

Obs	**Lunar feature measurement**	**Method**	**Calc. value**	**Actual value**	**Difference**
# 1	Crater Eratosthenes size	DTM	61.8 km	58 km	+ 7 %
# 1	Mons Pico height (1)	DTM	2,710 m	2,400 m	+ 13 %
# 2	Crater Bullialdus size	DTM	69.0 km	60 km	+ 15 %
# 2	Mons west to Bullialdus heigth	DTM	1,550 m	?	?
# 3	Crater Gassendi size (1)	DTM	115.0 km	101 km	+ 14 %
# 3	Promontorium Kelvin height (1)	DTM	1,900 m	1,890 m	+ 1 %
# 4	Crater Ptolemaeus size	PM	145.0 km	164 km	- 12 %
# 4	Crater Alphonsus size	PM	110.6 km	108 km	+ 2 %
# 4	Crater Plato size	DTM	118.0 km	109 km	+ 8 %
# 4	Mons Pico height (2)	PM	2,310 m	2,400 m	- 4 %
# 4	Mons Pico height (3)	DTM	2,170 m	2,400 m	- 10 %
# 5	Promontorium Kelvin height (2)	DTM	1,840 m	1,890 m	- 3 %
# 5	Crater Gassendi size (2)	DTM	109.1 km	101 km	+ 8 %
# 6	Crater Gassendi size (3)	DTM	109.4 km	101 km	+ 8 %
# 6	Crater Gassendi size (4)	PM	111.4 km	101 km	+ 10 %

The diameter of the crater Gassendi was measured three times applying the *drift timing method* and once using the *photographic method*. In the *DTM* the final results were 115.0, 109.1 and 109.4 km, which averages 111.2 km, being *10 % greater than the actual value* and almost the same as the value obtained through the *PM* (111.4 km). In this case, through the independent application of both proceedings *it has been achieved the same result.*

All remaining diameter values obtained by applying the *drift timing method* were also over-estimated (7 % in the case of Eratosthenes, 15% for Bullialdus, and 8% for Plato). One possible explanation of the systematic over-measured error in the diameter size could have been selecting the borders of the beginning and ending of the diameter of the craters *far beyond its real locations*. Anyway the final precision of *15 % is within the expected accuracy.*

The height of the Mons Pico also was measured applying both methods, obtaining 2,710 and 2,170 m by the *DTM* (averaging 2,440 m) and 2,310 m by the *PM*. The first *DTM* measure (2,710 m) was obtained at the waxing gibbous phase, that is with the projected shadow *towards the west*, and the second *DTM* measure (2,170 m) was obtained at the last quarter phase, that is with the projected shadow *towards the east*. As the Sun's altitude at the moment of both measurements were almost the same (6.1° and 5.4°) that fact do not justify the final result differences. If the *apparent flat horizontal surrounding floor* (30 km around the Mons Pico) actually has a non-zero slope in the east-west direction (despite being at the border of the Mare Imbrium), that could be the main reason of the difference in between the two measures.

Comparing the two measures of the Mons Pico height obtained in the same Observation # 4 by the two different methods (2,310 m by the *PM* and 2,170 m by the *DTM*), *the 6 % difference is quite satisfactory*.

The height of the Promontorium Kelvin was also twice measured by the *DTM*, each time with its shadow projected towards opposite directions, *achieving almost identical results*.

As a final evaluation of the experimental results obtained in this project by the application of the *photographic method* and the *drift timing method* to the same lunar features in the same conditions, the conclusion is that both proceedings have achieved almost the same values, with an acceptable accuracy of 15 %.

In order to improve the precision of the observational measurements, the first obvious suggestion is to use a well-calibrated astrometric or X-Y positioning micrometric eyepiece instead of fixed crosshair eyepiece, and secondly to use more magnification.

Any observational measurement includes appreciation errors and timing measurements also implies reaction errors. Both experimental errors become reduced as the measured size of the object seen at the eyepiece increases its image dimensions.

The use of accurate data for the ephemerides and features coordinates naturally minimizes errors introduced through the calculations. Lastly, the calculation for the height measurement can be optimized through the use of the "*improved method of projected shadows*" by William Davis, a theoretical development more accurate than the simplified version herein applied.

Conclusions

The observational direct measurement of lunar surface features is an enjoyable task that can be executed by means of usual astronomical equipment, potentially achieving satisfactory results if a minimum of care is observed along the overall proceeding.

After any observational applied procedure it will be necessary to process the initial features measures, incorporating the corresponding Moon data (ephemerides, coordinates) valid for the same time of the measurements and as precise as possible. All the required information is available at the internet.

The expected accuracy of about 15 % for the accomplished values can even be improved.

Acknowledgement

The author wishes to thanks the practical advices about the drift timing method received from the amateur astronomer Harry Jamieson (creator of the well-known Lunar Toolkit software) and also for sending him a copy of the MacDonald's referenced 1931 paper.

References

a) Measurement of lunar surface sizes
"Lunar Crater Project", http://paganastronomy.net/lunarcrater.html

"Plate Scale", http://virtual.parkland.cc.il.us/AST102/labs/help/platescale.html

"Diameter & depth of lunar craters", Andre James Clayden, Springbrook Research Observatory, http://maguires.com/astronomy/lunar_craters2.htm

b) Measurement of lunar surface heights
"The Mountains of the Moon", Bill Davis, Sky & Telescope magazine, Nov 1998 issue, pages 114-118

"On the Determination of Relative Lunar Altitudes", Thomas L. MacDonald, Journal of the British Astronomical Association, volume 41, June 1931, pages 367-379

"Determination of lunar elevations by the shadow method: analysis and improvements", William F. Davis III, American Lunar Society, http://otterdad.dynip.com/als/page37.html

"Observing the Moon", Gerald North, Cambridge University Press, 2000, pages 147-150

c) Moon's ephemerides
http://www.xylem.demon.co.uk/kepler/jsmoon.html
http://sunearth.gsfc.nasa.gov/eclipse/TYPE/moon3.html

d) Lunar features coordinates
http://www.fourmilab.ch/earthview/lunarform/lunarform.html
http://cass.jsc.nasa.gov/pub/research/lunar_orbiter/index.html

e) Promontorium Kelvin height
"Exploring the Moon through binoculars and small telescopes", Ernest H. Cherrington, Jr., Dover Publications, Inc., 1984, page 198

f) Mons Pico height
http://www.astrosurf.com/lunascan/Picobk.htm